Research Report

CONTOUR COMPLETION AND RELATIVE DEPTH:
Petter's Rule and Support Ratio

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Abstract—*The ability to see complete objects despite occlusion is critical to humans' visual success. Human vision can amodally complete visual objects that are partially occluded, and modally complete visual objects that occlude other objects. Previous experiments showed that the perceived strength of a completed contour depends on its support ratio: the ratio of the length of the physically specified contour to the total length of the contour. Other experiments showed that human vision prefers to make modal completions as short as possible, an effect known as Petter's rule. The experiment reported here examined the relationship between Petter's rule and support ratio, showing that both affect modal completion in figures of homogeneous color, but that when they compete Petter's rule dominates. Finally, our results confirm that Petter's rule is an effect of relative gap lengths and not of relative size.*

Occlusion is a ubiquitous feature of the visual world. Most objects people see are partly hidden behind other objects. Despite this, people see them not as isolated fragments, but as complete, unitary objects. In Figure 1a, for example, the viewer sees a single cat behind a window pane, not four different cat pieces. And in Figure 1b, the unity of the partially occluded object is apparent even though the object is unfamiliar. This ability to see unitary objects despite occlusion is fortunate, because otherwise fragments and pieces would appear and disappear as people moved around, and humans' visual world would resemble the "blooming, buzzing, confusion" that William James described. Indeed, this ability is not unique to humans. All visual animals face the occlusion problem, and recent studies show that many readily solve the problem, including chicks (Lea, Slater, & Ryan, 1996; Regolin & Vallortigara, 1995), hens (Forkman & Vallortigara, 1998), and mice (Kanizsa, Renzi, Conte, Compostela, & Guerani, 1993).

The ability to complete partially occluded objects has been termed *amodal completion* (Michotte, Thines, & Crabbe, 1964/1991). The term amodal indicates that the completion has no sensory characteristics, such as a perceived brightness gradient. In Figure 1, for example, although one is aware of the unity of the objects behind the window panes, one does not see a brightness gradient, or contour, in the occluded regions.

In *modal completion* (Michotte et al., 1964/1991), which occurs when an object partially occludes a surface of the same color, one does see a brightness gradient along the completed contours. In Figure 2, for example, not only is one aware of completed triangular shapes that occlude the disks, but one also sees a clear difference in brightness between the inside and outside of the completed contours—even though there are, in fact, no gradients in any image property in those regions: In Figure 2a, the inside of the completed triangle looks whiter than the surrounding white; and in Figure 2b, the inside of the completed triangle looks blacker than the surrounding black. Such completed figures, and their associated contours, have been termed "anomalous," "illusory," or "subjective" (see, e.g., Petry & Meyer, 1987; Spillmann & Dresp, 1995).

SUPPORT RATIO

What determines whether human vision will interpolate between two given edges to construct a completed contour? One factor is that the two edges must be appropriately aligned. In Figure 3, for example, each of the "Pacman" shapes in Figure 2 is slightly rotated, and, as a result, one no longer sees the illusory figures that are so striking in Figure 2. Kellman and Shipley (1991) proposed a precise criterion for edge alignment, termed *relatability*. According to this criterion, the extensions of the two edges must meet, and their exterior angle of intersection must be acute (see Kellman & Shipley, 1991, pp. 174–176). Although relatability is an all-or-none criterion, it can be extended to a graded measure of edge alignment (Singh & Hoffman, 1999) that can better track the gradedness in psychophysical data.

Furthermore, Shipley and Kellman (1992) have proposed that the strength of an interpolated contour depends on its *support ratio*, that is, the ratio of the length of the physically specified contour (i.e., the contour specified by a luminance gradient) to the total length of the contour (physically specified plus interpolated). In a series of experiments, Shipley and Kellman (1992) demonstrated that the support ratio predicts perceived strengths of illusory figures. Figure 4a demonstrates the role of the support ratio. In each display, the length of the physically specified contour is $2r$, where r is the radius of the black disks in the display; the total length of a contour is l, the length of the side of the square in the display. Hence, the support ratio is $2r/l$. As predicted, the strength of the illusory square increases with r, and decreases with l (Shipley & Kellman, 1992).

An important property of support ratio is that it is scale invariant: It does not change if a display is uniformly blown up, or shrunk, so that any unit-formation processes that are based on support ratio will not change unit assignments as the viewer moves toward or away from a scene. By contrast, the length of an interpolated contour is not scale invariant: If the display shrinks to half its size, so does the length of the interpolated contour (see Fig. 4b). Hence, any theory of interpolation strength based on absolute length of the interpolated contour would predict that this strength would vary with the viewer's distance to the display. In their experiments, however, Shipley and Kellman (1992) found that the perceived strength of interpolation is largely independent of scale (but see Dumais & Bradley, 1976). You can see this by comparing Figure 4a with 4b. Hence, scale invariance is a desirable property of the support ratio.

PETTER'S RULE

Consider the display in Figure 5a, which consists of a single, irregular shape of homogeneous color. One sees this display not as a single

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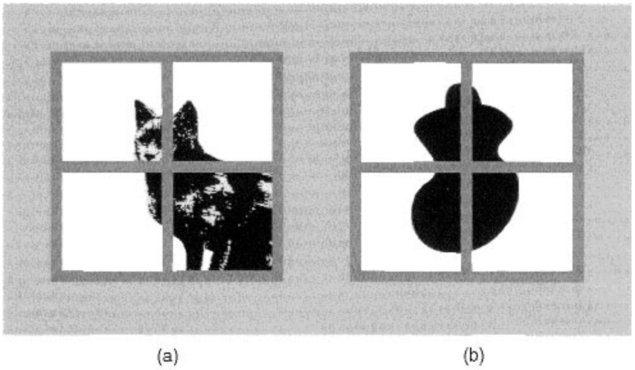

Fig. 1. Examples of amodal completion. An observer sees unitary objects, not isolated fragments, behind the window panes. Amodal completion is critical to inferring the unity of partially occluded objects.

object, but as two different objects—one occluding the other. The depth ordering of the two objects is ambiguous: One can see the horizontal shape modally completed in front (hence, with horizontal illusory contours) and the vertical shape amodally completed in back—or vice versa. This ambiguity can, of course, be removed by adding stereoscopic information, as in Figure 5b (see Anderson & Julesz, 1995, for a detailed theory of illusory contour formation in stereopsis). However, monocular geometric properties can also affect the perceived depth ordering in such figures of homogeneous color. In Figure 6a, for example, one sees the trapezoidal object modally completed in front, and the thinner object amodally completed in back—even though there are no T-junctions to signal occlusion. In Figure 6b, one sees two figures intertwining in depth (example based on Kanizsa, 1979). And in Figure 6c (painting by Kanizsa, 1979, p. 40), one sees the fishing rod pass behind the sailboat even though one knows that it should pass in front.

The examples in Figure 6 nicely demonstrate *Petter's rule* (Petter, 1956; Kanizsa, 1979) for modal completion of visual contours:

> Petter's rule: Human vision prefers to make modal completions (i.e., illusory contours) as short as possible.

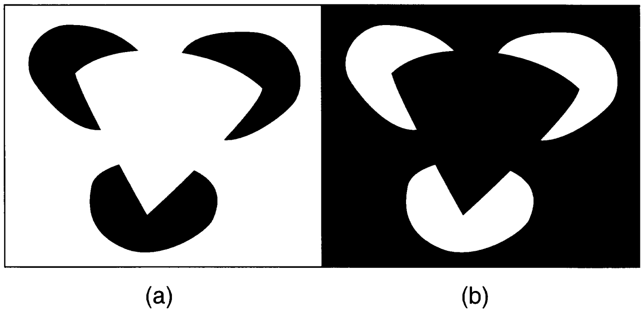

Fig. 2. Examples of modal completion. In each display, an observer sees a triangular shape overlying three blobs, rather than simply three "Pacman" shapes. Modal completion is critical to inferring the unity and shape of objects that are camouflaged by underlying surfaces with similar surface properties.

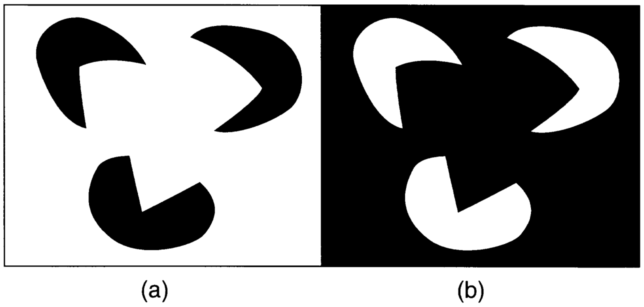

Fig. 3. Demonstration of the role of edge alignment in modal completion. The completion seen in Figure 2 is destroyed by rotating the "Pacman" shapes slightly.

Petter (1956) argued for this rule on the grounds that modal completion requires more "energy" than amodal completion. Other researchers have suggested that modal completion is more "expensive" (Tommasi, Bressan, & Vallortigara, 1995) or that modal completion has a higher threshold (Takeichi, Nakazawa, Murakami, & Shimojo, 1995) than amodal completion. The outcome, in either case, is that the figure (or portion of figure) requiring shorter interpolations to complete it is seen as being in front. Petter's rule applies not only to static patterns (Seyranian & Hoffman, 1998; Tommasi et al., 1995), but also to dynamic displays, such as moving plaids, in which it influences the perceived direction of motion (Bressan, Ganis, & Vallortigara, 1993; Vallortigara & Bressan, 1991). Of course, Petter's rule also interacts with other visual factors, such as stereo disparity and good continuation (Tommasi et al., 1995).

Some researchers have suggested that Petter's rule follows from the heuristic that larger figures should be seen in front—because, as a result of perspective projection, close objects tend to have larger retinal images (e.g., Stoner & Albright, 1993). According to this suggestion, the trapezoidal shape in Figure 6a is seen in front simply because it is larger than the other shape. However, Petter's rule is independent of the larger-is-closer heuristic because the length of interpolated contours can vary independently of figure size (Bressan et al., 1993; Tom-

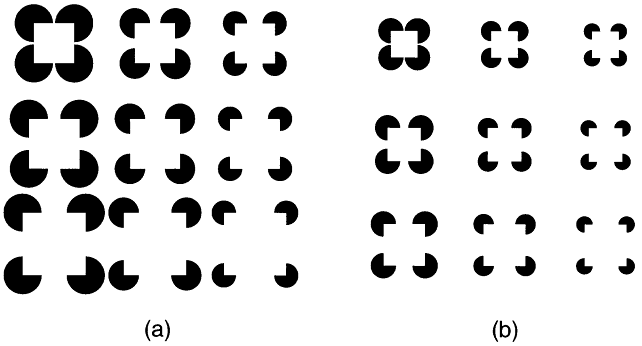

Fig. 4. Demonstration of support ratio and its scale invariance. When the displays in (a) are uniformly shrunk to half their sizes, as shown in (b), the support ratio associated with each modally completed segment remains unchanged. This uniform scaling does not affect the perceived strength of completion (see Shipley & Kellman, 1992).

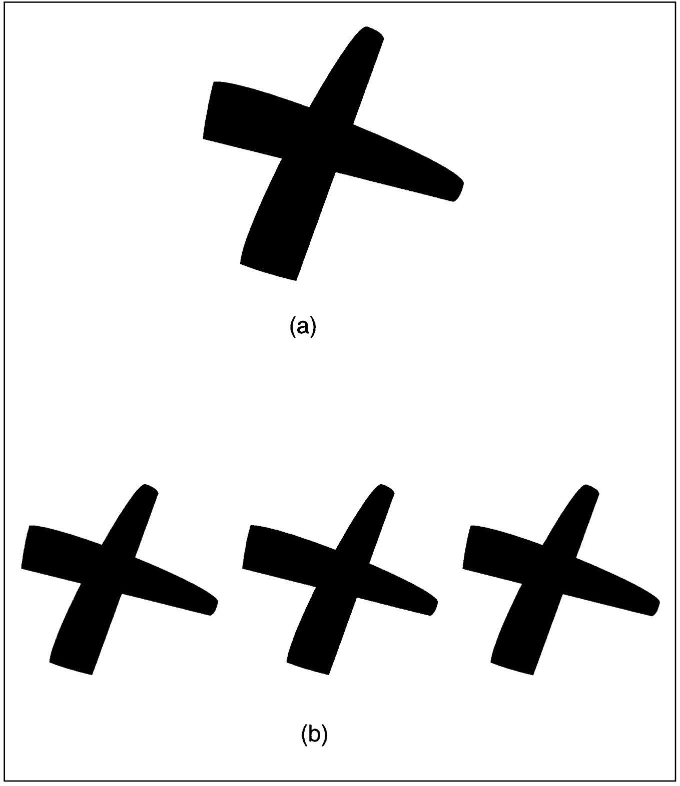

Fig. 5. Relative depth and modal contours in figures of homogeneous color. Even though the figure in (a) contains no T-junctions to signal occlusion, it is seen as two objects, one occluding the other, and not as a single cross-shaped object. But the depth ordering is ambiguous: One can see either the vertical or the horizontal bar as modally completed in front. Adding stereoscopic information, as in (b), can resolve this ambiguity.

masi et al., 1995). Indeed, Tommasi et al. (1995) gave examples in which the same two homogeneously colored figures, in different relative positions, can induce opposite orderings of perceived depth. Furthermore, as we discuss shortly, one can pit Petter's rule against the larger-is-closer heuristic, and Petter's rule wins. Hence, the larger-is-closer heuristic fails to provide an ecological motivation for Petter's rule. However, Petter's rule can be derived naturally from properties of transversal intersections of generic shapes in three dimensions (Singh, Seyranian, & Hoffman, 1999).

PETTER'S RULE OR THE SUPPORT-RATIO RULE?

Consider Figure 6. Note that, in each display, the contour that is modally completed, and seen in front, is also the one with the higher support ratio. Support ratio, as we saw earlier, directly affects the strength of contour completion; and if modal completions are more expensive than amodal completions, this suggests that contours with higher support ratios should be seen as modally completed. This line of reasoning leads to a rule, different from Petter's, that explains the perceived relative depths in these displays:

The support-ratio rule: Human vision prefers modal completions with higher support ratios.

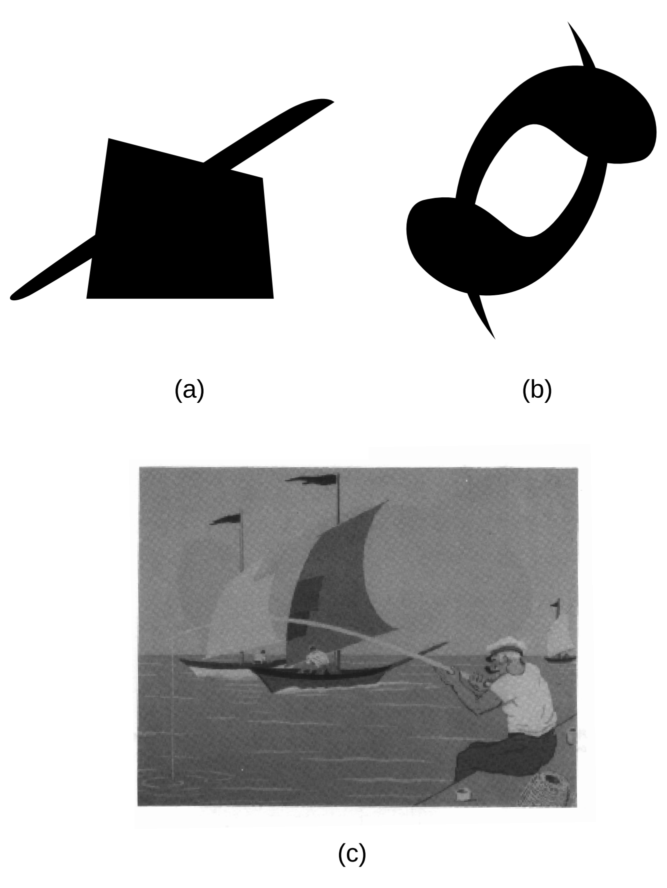

Fig. 6. Demonstrations of Petter's rule. Despite the lack of stereo information and T-junctions, the depth ordering is perceived unambiguously in these displays. In (a), the trapezoidal shape is seen in front. In (b), the two shapes are seen intertwining in depth (example based on a figure by Kanizsa, 1979). In (c), the fishing rod is seen as passing behind the sailboat, even though this leads to a paradoxical percept (painting by Gaetano Kanizsa; reproduced with permission from Kanizsa, 1979, p. 40).

This rule and Petter's rule provide differing explanations for perceived relative depth in figures of homogeneous color. Furthermore, they are both, as one would like, scale invariant. We have already noted that the support ratio is scale invariant. Petter's rule is also scale invariant because it involves relative lengths. Albert (1995) hypothesized that the stronger interpolated contour becomes the modal contour in stimuli of homogeneous color, such as Figure 6, that are perceived as overlapping surfaces. If this hypothesis is true, then Petter's rule and the support-ratio rule would sometimes make different predictions about depth order in these stimuli.

The primary difference between Petter's rule and the support-ratio rule is that the latter uses the length of the supporting edges, but Petter's rule does not—it uses only the relative lengths of the gaps to be interpolated. In the experiment we report here, we studied the relative contributions of Petter's rule and the support-ratio rule in determining modal completions and relative depths in figures of homogeneous color.

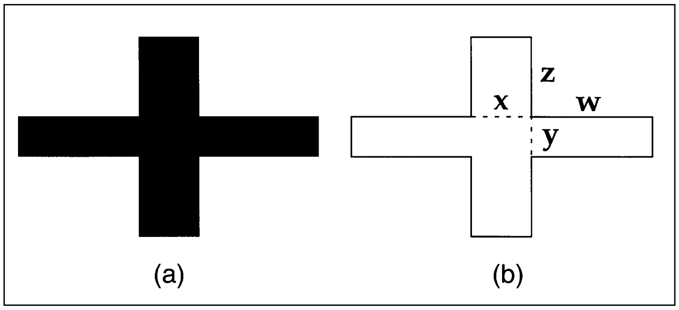

Fig. 7. Example of the shapes used in the experiment (a) and the variables used to parametrize them (b).

EXPERIMENT

We used, for simplicity, cross-shaped figures that were symmetric about their horizontal and vertical axes (see Fig. 7a). Such figures are easily parametrized by four quantities (x, y, z, and w in Fig. 7b). The two variables relevant to this study are

- *distance ratio* (D) = length of horizontal completion/length of vertical completion = x/y
- *support ratio* (S) = support ratio for the horizontal completion/support ratio for the vertical completion = $[2w/(x + 2w)]/[2z/(y + 2z)]$.

As D increases, the length of the horizontal completion grows relative to the length of the vertical completion, and Petter's rule predicts that the proportion of times the vertical bar is seen in front should increase. As S increases, the support ratio for the horizontal bar grows relative to the support ratio for the vertical bar, and the support-ratio rule predicts that the proportion of times the vertical bar is seen in front should decrease. Hence, by creating a factorial design in D and S, we were able to study the interaction between Petter's rule and the support-ratio rule. Note also that because D and S are ratios, they are both scale invariant.

Method

Subjects

The subjects were 12 students, who volunteered to participate for extra credit toward their psychology courses at the University of California, Irvine.

Stimuli

The stimuli were the nine shapes shown in Figure 8.

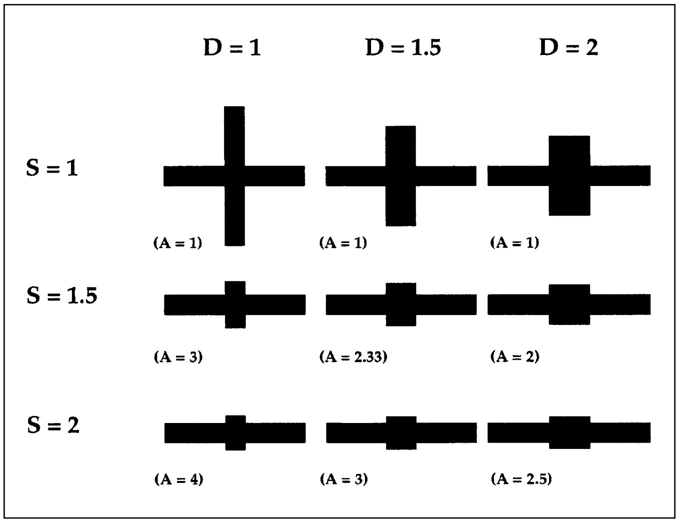

Fig. 8. The nine stimuli used in the experiment and their associated values of distance ratio (D), support ratio (S), and area ratio (A). These stimuli were presented to the subjects rotated either +45º or –45º.

Design

The independent variables were distance ratio (D) and support ratio (S). Each variable had three levels: 1, 1.5, and 2. Both variables were run within subjects. Each stimulus was presented 16 times, eight times rotated +45º and eight times rotated –45º in order to minimize any effects of orientation. This resulted in a total of 144 experimental trials per subject. The experimental trials were preceded by 18 practice trials. The dependent measure, V, was the proportion of times that the "vertical" bar[1] was seen in front.

Apparatus and procedure

The figures were displayed on a 1024 × 768 monitor by a Power Macintosh G3 computer using the SuperLab program. Subjects used a keyboard to respond.

Subjects were seated 0.5 m from the computer monitor. They were instructed that, on each trial, they would see two rectangular bars, one in front of the other and partially hiding it. They were to judge, as quickly and carefully as possible, which of the two bars was in front.

Each trial was structured as follows: First, one of the crosses (see Fig. 8) was presented for 2 s, rotated at either +45º or –45º. During this time, the subject was to decide which bar he or she saw in front. After a 500-ms blank interval, the same cross appeared as before, with the two bars that constituted the cross presented below it. The two bars were labeled "a" and "b," respectively, with the labeling counterbalanced across trials. The subject had to indicate, by pressing the appropriate key ("a" or "b"), which of the two bars appeared to be in front. This response terminated the trial.

Results and Discussion

Figure 9 shows the results of the experiment. The proportion of times the vertical bar was seen in front increased with distance ratio, $F(2, 22) = 12.491$, $p < .0005$, and decreased with support ratio, $F(2, 22) = 6.158$, $p < .01$. This pattern shows that both Petter's rule and the support-ratio rule affect modal completion and relative depth in figures of homogeneous color. The interaction between the two variables, however, was not significant, $F(4, 44) = 0.465$, $p > .75$; the two combine additively.

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[1] By "vertical," we mean the direction defined by the bar whose length is given by $2z + y$ (see Fig. 7). The stimuli in Figure 8 are shown vertically aligned according to this convention.

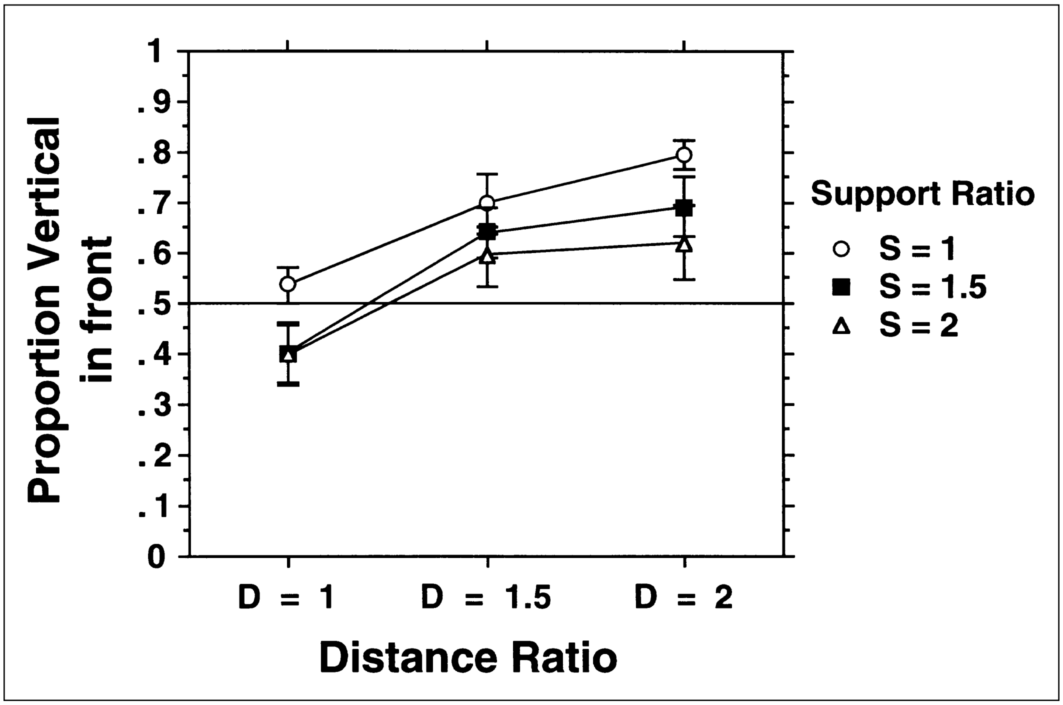

Fig. 9. Results of the experiment. Proportion of times the vertical bar was judged to be in front is plotted as a function of distance ratio and support ratio.

Consider again the stimuli for the experiment (Fig. 8). Note that for each stimulus, the vertical completion is shorter than (or, if D = 1, equal in length to) the horizontal completion; also, the vertical completion has a support ratio lower than (or, if S = 1, equal to) the horizontal completion. This means that, for a given stimulus, if subjects see the vertical bar in front in more than 50% of the trials, they prefer to see shorter modal completions; if they see the vertical bar in front in less than 50% of the trials, they prefer to see modal completions with higher support ratios. In Figure 9, seven of the nine data points lie above the 50% line, and the only two data points that lie below the 50% line have distance ratios of 1 (i.e., the vertical and horizontal completions have equal lengths, so Petter's rule does not apply). Hence, when Petter's rule and the support-ratio rule compete, subjects show a strong preference for Petter's rule.

To study the role of the larger-is-closer heuristic, we replotted the data (see Fig. 10) in terms of the distance ratio, D, and the area ratio, A, which is given by

$$A = \text{area of the horizontal bar/area of the vertical bar} = y \cdot (x + 2w)/x \cdot (y + 2z).$$

Multiple regression analysis revealed that the coefficients for both D and A were significantly different from zero ($V = 0.424 + 0.206D - 0.061A$, $R = .54$). Note, however, that A is 1 for stimuli with S of 1, and A is greater than 2 for all other stimuli (see Fig. 8). This means that if subjects preferred to see larger areas in front, they should have seen the horizontal bar in front more often than the vertical bar. But this happened only for two of the nine stimuli—both of which have a distance ratio of 1 (so that Petter's rule is neutral in these cases). Thus, although both distance ratio and area ratio affected subjects' responses, when Petter's rule and the larger-is-closer heuristic competed, Petter's rule again won.

CONCLUSIONS

Two rules predict the construction of modal contours and relative depth in figures of homogeneous color. Petter's rule predicts that human vision prefers to make modal completions as short as possible; the support-ratio rule predicts that human vision prefers to make modal completions with the highest support ratio. We tested the relative contributions of Petter's rule and the support-ratio rule, and found that both contribute to the perception of modal contours and relative depth. However, Petter's rule dominates when the two compete. Moreover, Petter's rule works even when it contradicts the larger-is-closer hypothesis, confirming that it is not merely an effect of perspective or interposition. Recent work shows that Petter's rule also works in other species besides humans, such as domestic hens (Forkman & Vallortigara, 1998).

These results are similar to those of another study (Singh et al., 1999) in which we found that distance ratio consistently predicts how subjects parse silhouettes into parts, but that area ratio does not. Together, the studies support the hypothesis that the same initial mechanisms are responsible for decomposing silhouettes into parts and for constructing modal completions in figures of homogeneous color.

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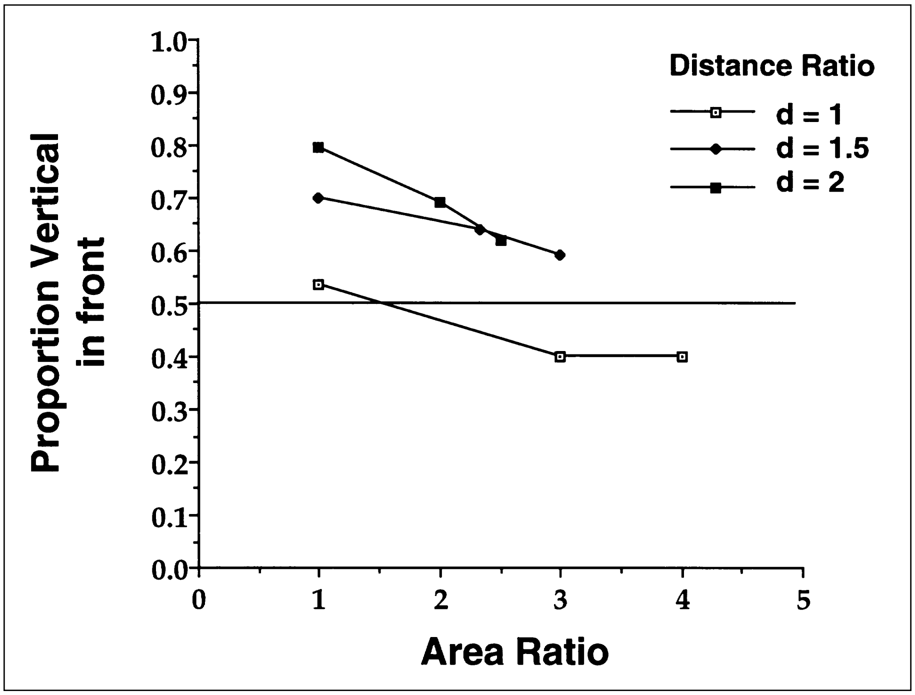

Fig. 10. Results replotted in terms of distance ratio and area ratio.

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