THE VISIBLE PERSISTENCE OF STIMULI IN STROBOSCOPIC MOTION

Joyce E. Farrell,* M. Pavel† and George Sperling

New York University, Washington Square, New York, NY 10012, U.S.A.

Abstract—This paper reports an improved paradigm to measure visible persistence. The stimulus is a pair of lines stroboscopically displayed in successive positions moving in opposite directions. The subjects' judgement of simultaneous appearance of all the presented lines is used to estimate visible persistence. This paradigm permitted independent manipulation of spatial and temporal stimulus separations in linear motion. The resulting estimates of visible persistence increase with spatial separation up to 0.24 deg of visual angle and approaches a maximum value at larger spatial separations. The results are consistent with the existence of a hypothetical visual gain mechanism that operates over small retinal distances to effectively decrease persistence duration with decreasing spatial separation.

Visible persistance   Stroboscopic motion   Apparent motion

INTRODUCTION

Stroboscopic motion

In artificial representations of natural object motion, such as in movies, television, and computer driven visual displays, continuous motion is represented by a succession of discrete samples. By increasing the temporal sampling rate of an object moving at a fixed velocity, one can create an illusion of motion that is indistinguishable from the appearance of continuous motion. (Sperling, 1976; Watson, Ahumada & Farrell, 1983). When the sampling rate is not high enough, however, the appearance of continuous motion is replaced by multiple images of the moving object.

Consider, for example, the stroboscopic representation of a single vertical line moving horizontally across a display screen. For some spatial and temporal separations of the line in stroboscopic motion, instead of a single line, observers perceive a number of lines moving together across the screen (Allport, 1968). An analogous phenomenon in real motion is the apparent elongation of a rapidly moving object (Newton, 1720; Allen, 1926). The obvious explanation for the apparent multiple lines in

stroboscopic motion and the smearing in real motion is that each flash of the line produces an image whose visibility persists over time and which, therefore, temporally overlaps subsequent flashes of the line.

According to this explanation, the visible persistence of an image can be estimated by the number of successive stimuli that appear to be simultaneous. For example, if a stimulus is visible for approx. 100 msec, it should appear to temporally overlap stimuli that follow in less than 100 msec. Previous estimates of the duration of visible persistence based on this method range between 100 and 300 msec (Coltheart, 1980). When the distance and time between successive stimuli approaches zero, as in the case of real motion, the duration of visible persistence can be estimated by the length of an object's blur streak. Estimates of the duration of visible persistence based on this latter method (Burr, 1980) range between 2 and 5 msec. Apparently, the procedure for investigating the persistence of stroboscopically moving stimuli generates a different estimate of persistence duration than the procedure for investigating the persistence of continuously moving stimuli. But should we attribute this difference to differences in the paradigms used for estimating persistence duration? Or do different perceptual mechanisms underlie the visible persistence of stimuli in stroboscopic ("apparent motion") and continuous ("real") motion?

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*To whom reprint requests should be addressed, present address: Hewlett-Packard Laboratories, P.O. Box 10490, Palo Alto, CA 94303-0971, U.S.A.
†Present address; Department of Psychology, Stanford University, Stanford, CA 94305, U.S.A.

Farrell (1984) estimated the visible persistence of stimuli in stroboscopic motion by asking observers to report the number of successively presented stimuli that appeared to be simultaneously visible. She found that the estimated durations of visible persistence increased with the distance separating the successive stimuli. This finding, taken together with reports by Dixon and Hammond (1972), Allport (1970) and DiLollo and Hogben (1985), provides an explanation for the paradox that the visible persistence of continuously moving stimuli is relatively short (Burr, 1980) when compared to the persistance of stroboscopically moving stimuli (Allport, 1970; Efron & Lee, 1971). When the distance between successive stimuli is small, the duration of visible persistence is small; as the distance increases, persistence increases. This reduces the smear generated by moving objects but extends the time available to process stationary objects (e.g. Burr, 1980; DiLollo, 1980; Sperling, 1967).

Because visible persistence can have many different causes, it is important to determine whether lawful behavior measured using one paradigm extends to other procedures. In this paper, we first review some previous methods for estimating visible persistence. We then describe a new procedure that we believe overcomes some of the limitations of the previous procedures. Using our new method, we extend the measurements made by Farrell (1984) and by DiLollo and Hogben (1985) by investigating the duration of visible persistence over a wide range of spatial separations. The new data that we report in this paper sheds light on the type of mechanism that may underlie the visible persistence of moving stimuli and the range over which the mechanism operates.

Paradigms for estimating the duration of visible persistence

The duration of visible persistence of an object in stroboscopic motion can be estimated by the number of successive objects that appear to be physically present at the same time (Allport, 1968; Dixon & Hammond, 1972; Efron & Lee, 1971). Here, we consider the hypothesis that for describing the appearance of stroboscopically moving objects, the visual system can be represented by two stages. The first stage represents low level perceptual units and is represented by a spatio-temporal filter whose response embodies visible persistence; it lengthens the duration of its visual inputs. The second

stage monitors the perceptual units of the first stage and decides which of the units are active by comparing their output to a threshold. The number of simultaneously active units corresponds to the number of simultaneously visible stimuli. For example, suppose that a briefly presented luminous line elicits a visual sensation (the first stage response) that decays; and, after 100 msec, the persisting sensation is no longer visible (below threshold of the second stage). Suppose also that the line is presented every 100 msec in a new position, as illustrated in Fig. 1. This system will report that it sees only one line because the visible persistence of successive stimuli does not overlap. When the line is represented every 50 msec, the system reports seeing two lines because the visible persistence of two successive stimuli will overlap. By the same reasoning, the system will report 3 lines when the line is presented every 33 msec and 4
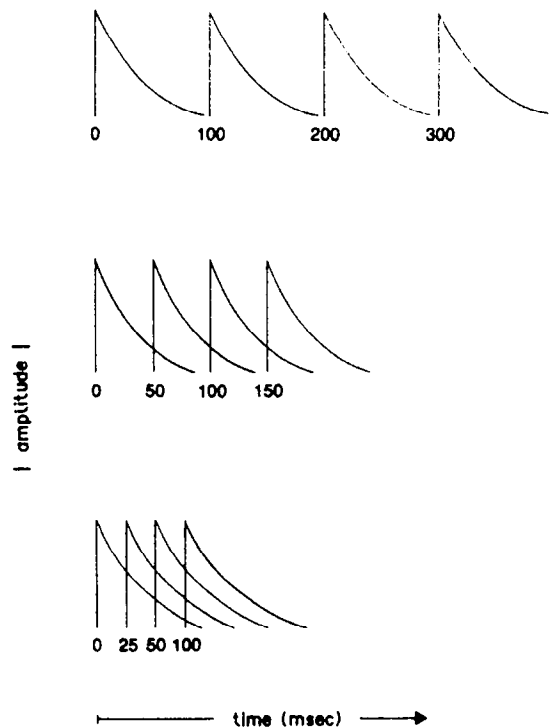

Fig. 1. This figure illustrates the hypothetical case in which a briefly presented visual stimulus creates a persisting sensation that decays over time such that after 100 msec the persistence decays to a level below which it is no longer visible. In the top panel, the stimulus is presented in a new position every 100 msec and a single line should appear to be present at any one instant in time. The second and third panels show instances in which successively presented stimuli generate visual responses that overlap in time. In general, if the perceived number of stimuli increases linearly with the rate of stimulus presentation, then the slope of the linear function can be used to estimate the duration of visible persistence.

lines when the line is presented every 25 msec. In general, when the perceived number of lines increases linearly with the rate of stimulus presentation, then the slope of the linear function can be used to estimate the duration of visible persistence.

Allport (1968, 1970) and Efron and Lee (1971) estimated the duration of visible persistence from the number of simultaneously visible lines by means of a computation very similar to that embodied by the 2-stage system described above. For example, Efron and Lee (1971) assumed that visible persistence can be described by a single real number, its duration p. Efron and Lee reasoned that the number of stimuli that will appear to be simultaneous is $n = p/t$ where t is the time interval separating two adjacent stimuli, and n is the average number of observed lines. Implicitly, this prediction assumes that the probability that the number of successive stimuli will appear to be simultaneously visible is proportional to the degree to which the visible persistence of successive stimuli overlap. Let the number of lines simultaneously observed on a particular trial be a random variable N and let n be the expected value of N. These assumptions lead to the prediction that:

$$n = E(N) = \max\left(\frac{p}{t}, 1\right).$$

When $p \leqslant t$, the expected value of N, $E(N)$, is 1 representing the fact that observers report seeing a stimulus even when it is not visible all the time. When $p > t$, $E(N)$ is p/t. This prediction is precisely correct only for integer values of p/t (see below).

Efron and Lee (1971) varied the rate at which a rotating line was strobed and asked observers to report how many lines they saw at any one time. They derived the duration of visible persistence from the slope of the linear functions relating the strobe rate and the number of lines observers reported. Estimates of the duration of visible persistence ranged between 133 and 144 msec.

The most significant difficulty with these procedures for estimating visible persistence is that the observer must count the number of perceived lines. To determine the visible persistence of stroboscopic stimuli that approximate real motion, we must estimate the persistence of closely spaced stimuli. This requires counting a large number of closely spaced lines, where both

the spacing and the number make counting impractical. Alternatively, the classical procedure (Newton, 1720; Allen, 1926) for estimating persistence of an object in real motion (revived by Burr, 1980) utilizes the length of the object's blur streak to estimate visual persistence. While it avoids the counting problem, this method still requires the subject to estimate the size of a rapidly moving object.

A second problem occurs when the spatial position of the stimuli in stroboscopic or real motion is uncertain. In this paradigm (Efron and Lee, 1971) the experimenter has no control over where or when the count of visible lines occurs. Further, the experimenter does not known during what fraction of the trajectory the reported number of lines is visible.

Third, the observed duration of persistence and the number of simultaneously visible stimuli are not absolutely constant from trial-to-trial but, like everything else psychologists measure, vary. The stochastic nature of these measures must be reflected in the data collection and analyses procedures. Thus, the observed duration of visible persistence should be represented by a random variable. The explicit treatment of persistence as a random variable in data analysis, and the measurement of its distribution may prove useful for evaluation of potential theories.

We propose here a paradigm and a method of analysis to overcome the problems of counting, of spatial indeterminacy, and of measuring the random variation of persistence. The paradigm is used to extend the range of spatial and temporal conditions over which it has been possible to measure persistence in stroboscopic motion. The analysis is used to obtain estimations of the complete trial-to-trial distributions of persistence in the various conditions.

The paradigm

In our experiments, two vertical lines one above the other, move horizontally in stroboscopic motion in opposite directions over a fixed distance (Fig. 2). Successive positions, are separated by a fixed interval of time Δt and a displacement of Δx to the right for one line and $-\Delta x$ (leftward) for the other. For different Δt and Δx, observers report whether or not all the lines in both paths appear to be simultaneously present. They are instructed to respond "yes" if they perceive a flickering grating composed of all the positions of the lines and to respond "no" if they do not.

To estimate the duration of visible persistence with this paradigm, we assume that each briefly presented stimulus generates a visual response that decays over time. If the first presented stimulus in one row is still visible when the last presented stimulus occurs in the other row immediately above or below it, the observer responds "visible"; otherwise, "not visible". This paradigm determines the proportion of trials on which a stimulus remains visible from the first flash to the beginning of the last flash in a row.

Responses are inherently probabilistic. We assume that they reflect trial-to-trial variability in either or both the temporal waveform of the persistence response and in the subject's criterion for deciding whether the stimulus is visible. The analysis takes into account the probabilistic nature of the data in order to separate the effects of the retinal separation on (1) the mean duration of visible persistence and on (2) the trial-to-trial variation of visible persistence. The analysis does not distinguish between causes of variability, such as fluctuations in the underlying visual response and fluctuations in the threshold criterion.

EXPERIMENT 1

Method

Subjects. Data were collected from four observers, including one of the authors (JF). All observers had normal or corrected-to-normal vision.

Stimuli. The stimuli were vertical lines drawn on a HP1310 crt display with a P4 phosphor. The background of the display was illuminated by incandescent lights that produced a background luminance of $0.35 \, cd/m^2$. Subjects viewed the display from a distance of 94 cm and each vertical line subtended 0.235 deg of visual angle (0.386 cm). Each line was displayed for less than 1 msec at the same stimulus intensity. The horizontal and vertical distance between the centers of adjacent raster pixels was 0.0193 cm and each stimulus was composed of a vertical column of 20 raster pixels. Each pixel had a luminance directional energy (cf. Sperling, 1971) of 0.09 cd-sec. This stimulus intensity will hereafter be referred to as the *reference intensity*.

Two vertical lines were presented in a succession of positions, each position following the other by a fixed interval of time, Δt, and displaced to the right (or left) by a distance, Δx, as shown in Fig. 2. One of the vertical lines was
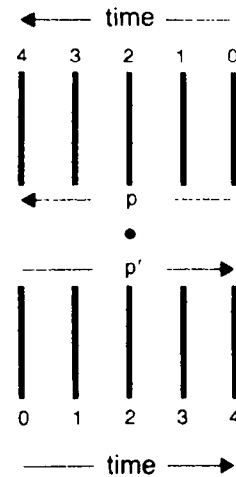

Fig. 2. The display for Expts 1–3: two vertical lines were presented in a succession of positions along the paths p and p' as shown above. Each position of the line followed the other by a fixed interval of time, Δt, and was displaced by a fixed distance, Δx, in a constant direction (left or right).

presented with its bottom 0.12 deg above a fixation point and extending upward for 0.24 deg. The other vertical line was presented symmetrically 0.12 deg below the fixation point. The two vertical lines were presented in the same horizontal positions, differing only in a spatial shift in the vertical direction and in the temporal order of presentation. On each trial, the direction of motion of the upper line was randomly chosen; the lower line moved in the opposite direction.

Subjects were instructed to stare at the fixation point for the duration of each stimulus presentation. The fact that the two vertical lines moved in opposite directions helped subjects to keep their gaze on the fixation point and discouraged them from tracking the stimulus with their eyes. Making any eye movement during the display would often cause it to appear distorted (see Farrell, Putnam & Shepard, 1984) and subjects quickly learned to suppress eye movements.

Across trials, stimuli differed in the distance between successive lines, Δx, the time interval separating the successive lines, Δt and the total number of lines that were presented, N. The distance, Δx, separating successive positions of each vertical line was either 0.12, 0.18 or 0.36 deg of visual angle. The length of the horizontal path of each vertical line was equal to the product of $(N - 1)$ and Δx. For example, when Δx was 0.12 deg of visual angle, N, was 13, 16 or 19 in order to obtain path lengths corresponding to 1.44, 1.80 and 2.16 deg, respectively. Similarly, when Δx was 0.18 deg, N was 9,

11 or 13 for the three respective path lengths. And when Δx was 0.36 deg, N was either 5, 6 or 7 for path lengths equal to 1.44, 1.80 and 2.16 deg, respectively. The stimuli were presented in separate blocks of 120 trials for each condition of path length.

Procedure. The subject initiated a trial by pressing a response key. After 600 msec, two vertical lines were presented in a succession of positions, one line beginning from the left of the fixation point and proceeding to the right and the other line beginning from the right of the fixation point and proceeding to the left. At the end of each trial, the subject pressed one of two response keys to indicate whether or not all successive presentations of the lines on both trajectories appeared to be simultaneously visible. Subjects were specifically instructed to respond "yes" if they perceived a flickering grating composed of all the positions of the lines above and below the fixation point and to respond "no" otherwise.

An experimental session consisted of three blocks of 120 trials corresponding to the three different path length conditions. Within each block of trials, each condition of spatial separation Δx was presented 40 times. The 40 repetitions were presented within two interleaved staircases. The total 120 trials resulting from the product of the 3 Δx, the 20 repetitions per Δx, and the 2 staircase conditions were presented in a random order.

The interstimulus interval was controlled by a modified up-down staircase (Levitt, 1970). The starting value of the interstimulus interval (ISI) in the first experimental session was 50 msec. If the subject responded "no", the Δt was decreased by 2 milliseconds and this new Δt was stored for the next presentation of this staircase. If the subject responded "yes" for *two* presentations of the same stimuli, the ISI was increased by 2 msec and this new ISI was stored to be presented later in the pre-arranged random sequence of trials. The staircase procedure adjusts the temporal separation so that 71% of the time the N successively presented stimuli appear to be simultaneously present. This same procedure was repeated for another interleaved staircase. The complete set of data provided by the two interleaved staircases allows us to estimate psychometric functions for each condition of spatial separation.

In subsequent experimental sessions, the initial value of the Δt was set equal to the estimated 71% threshold from the earlier sessions plus or minus a random number between 1 and 10 msec. The Δt in subsequent sessions was increased or decreased by a number that was proportional to the slope of the estimated psychometric function to insure that the psychometric function was sampled by at least 4 equally-spaced intervals.

All subjects participated in a minimum of 3 experimental sessions: one subject completed 6 sessions, two subjects completed 4 sessions and one subject completed 3 sessions.

Results and discussion

Method of analysis. Our analysis rests on the assumption that a briefly presented luminous line generates a visual response that decays over time, and that after some time the visual response reaches a threshold below which it is no longer visible. We make no assumption about the shape of the visual response; we simply assume that as long as the visual response generated by the stimulus is above threshold, the stimulus will appear to be present. If subjects report that all N lines appear to be simultaneously present, then we assume that, for some instant during that particular trial, the visual responses generated by the N lines were all above threshold. As a practical matter, from the subject's point of view, the question of N visible lines reduces to the simultaneous visibility of the first and last line. No subject reported that the first and last lines of a trajectory were visible but some interior line had vanished.

Let the observable time interval during which the image of all N lines are visible (i.e. above threshold) be a random variable, D. As noted earlier the random variability in D may be the result of threshold variability in the decision stage, variability of the decay function, or other random effects (noise). At the outset, we assume the distribution of D to be normal with mean τ and variance σ^2. This assumption is directly tested in the process of data analysis. For given values of Δt and N, we wish to find $p\,(\Delta t, N)$, the estimate of the probability that the first and last lines will appear to be visible simultaneously. $p(\Delta t, N)$ is equal to the probability that the first line has not decayed below threshold during the time interval $(N - 1)\Delta t$ separating the onset of the first and last stimulus, i.e.

$$p(\Delta t, N) = Prob[D > (N - 1)\Delta t]$$

$$= 1 - \Phi\{[(N - 1)\Delta t - \tau]/\sigma\}; \qquad (1)$$

where τ is the mean duration of visible persistence over trials, σ is the standard deviation of the duration of visible persistence over trials, and $\Phi[\tau, \sigma]$ is a cumulative normal distribution with mean τ and variance σ^2.

Maximum likelihood estimates of τ and σ were computed for each subject and each Δx. The estimations were performed using the numerical procedure STEPIT (Chandler, 1965) to maximize the likelihood that the data were generated by equation (1). To see how well the estimated mean τ and variance σ of persistence duration represent the data, the estimates were used to predict the frequencies of "visible" responses for each subject in each condition of Δx for each individual Δt reached by the staircase. Each of the 12 estimated normal distributions effectively predict the response probabilities. We cannot reject the predictions on the basis of a basis of a χ^2 test at $p < 0.05$ for any subject in any stimulus condition.

Data for each condition and each subject are shown in Fig. 3. The estimated mean duration of visible persistence, τ, and the estimated standard deviation, σ, of persistence duration are plotted as a function of the spatial separation, Δx, for different values of N. There are several interesting aspects of the data. First, Fig. 3 shows that the mean duration of visible persistence increases with the distance separating the successive stimuli, Δx, for all four subjects. This result re-affirms the basic finding reported by (Farrell, 1984).

Second, Fig. 3 shows that the mean τ and standard deviation σ of the visible persistence duration generated by a briefly presented stimulus do not vary with the number of stimuli, N, that are successively presented. The mean and standard deviation depend only on the distance separating successive stimuli, Δx. This result is also consistent with previous findings. Efron and Lee (1972) and Farrell (1984) observed that the number of successive stimuli that appear
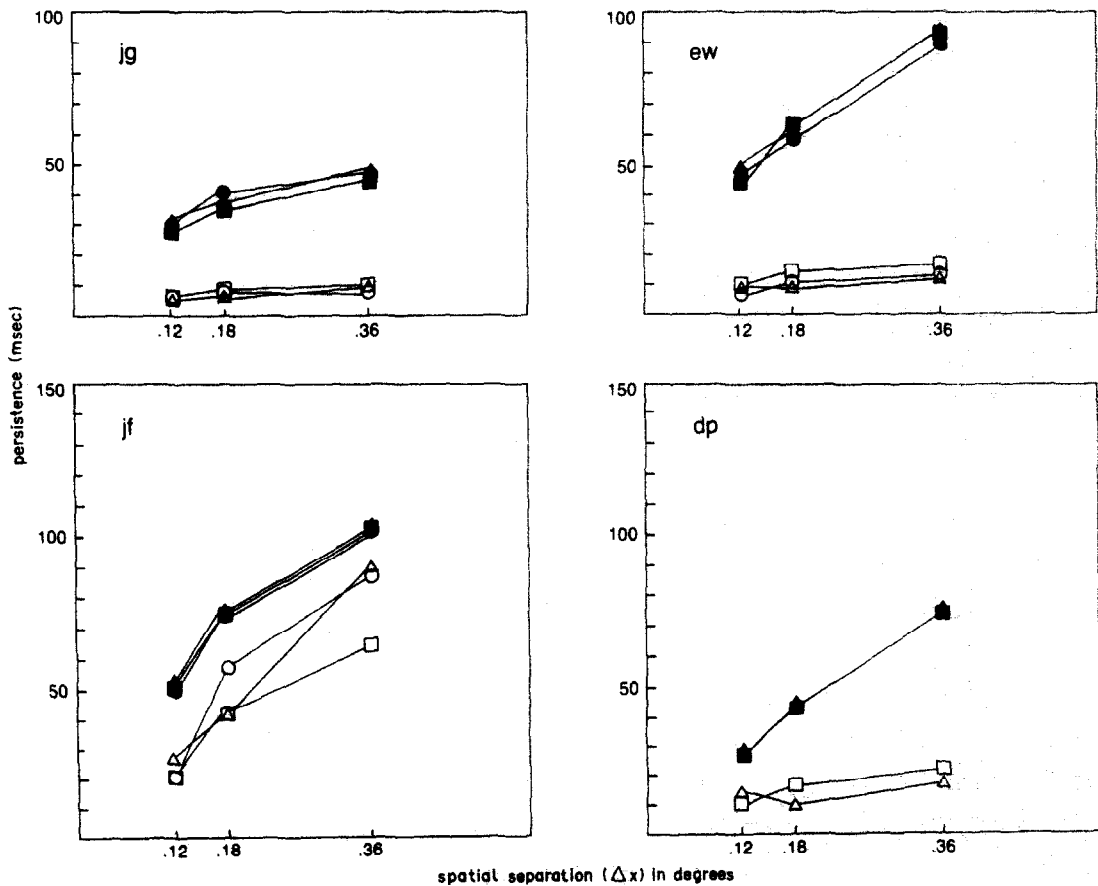

Fig. 3. The estimated mean (solid symbols) and standard deviation (open symbols) of the visible persistence of a briefly presented visual stimulus plotted as a function of the distance separating the stimulus line from other stimuli that occur later in time with the length of the stimulus path as the parameter. Circles, triangles and squares correspond to stimulus paths of 1.44, 1.80 and 2.16 deg visual angle, respectively. Each panel represents data from one subject.

to be simultaneously visible trades off with the temporal interval that separates successive stimuli.

The retinal eccentricity of each successively presented stimulus is proportional to $(N - 1)\Delta x$. Therefore, the invariance of persistence with N and, consequently, with eccentricity indicates that the duration of visible persistence does not vary with the eccentricities that were investigated (0.7, 0.9 and 1.1 deg). This result suggests that, over the local retinal region investigated, the duration of visible persistence is constant for a given spatial separation Δx. The result does not imply, however, that the retinal eccentricity of a stimulus might not influence the duration of visible persistence if it were varied over a wider range (cf. DiLollo & Hogben, 1985).

Finally, Fig. 3 shows that the variability of persistence duration increases with retinal separation for one of the four subjects (JF). As noted earlier, the individual differences in the variability of the duration of visible persistence across trials may reflect changes in the subjective threshold criterion or changes in the underlying visual response.

EXPERIMENT 2

In the previous experiment we found that for all subjects the mean duration of visible persistence increased with the distance separating the successive stimuli and, for one subject, the variability of persistence duration also increased with the spatial separation. This result is consistent with previous studies that used different experimental paradigms for estimating the duration of visible persistence (Allport, 1968, 1970; Efron & Lee, 1971). These previous studies have not reported limits to the increase of persistence duration with spatial separation. Nonetheless, it seems reasonable to assume that there is both a minimum and maximum duration of visible persistence. In order to place bounds on the increase in persistence duration with spatial separation, we conducted a second experiment and estimate the duration of visible persistence over a wider range of spatial separations.

Method

Subjects. The same four observers who participated in the first experiment (EW, DP, JG and JF) served as subjects in this experiment.

Stimuli. As in the previous experiment, the stimuli differed in the distance between

successive lines Δx the time interval separating the successive lines Δt and the total number of lines that were presented, N. The number of lines (N) was 25, 13, 9, 7, 5, 4 or 3 for Δx corresponding to 0.06, 0.12, 0.18, 0.24, 0.36, 0.48 or 0.72 deg visual angle, respectively. The lines were displaced over a total path length of 1.44 deg. All other aspects of the stimuli were identical to Expt 1.

Procedure. Each experimental session consisted of two or three blocks of 280 trials. Within each block of trials, each Δx was presented 40 times. The 40 repetitions were separated into two staircase conditions. The 280 trials were arranged in a random order of presentation.

One observer viewed 6 blocks of trials in two separate experimental sessions, another observer viewed 4 blocks of trials in two separate sessions, and two observers viewed 3 blocks of trials in a single experimental session. Observers rested between blocks of trials.

As in the previous experiment, two interleaved random staircases were used to distribute the data around a 71% threshold criteria. Depending on the subjects response, the temporal separation was adjusted such that 71% of the time the N successively presented stimuli appeared to be simultaneously present. The complete data set can then be used to estimate psychometric functions for each condition of spatial separation.

Results and discussion

As in the previous analysis, we assume that the probability that observers will report that the N successive lines appear to be simultaneously present is given by equation (1). Again, using the maximum likelihood procedure, we estimated the values of τ and σ that maximized the match between the predicted and the observed response probabilities for each observer, Δx, and for all values of Δt reached by the staircases.

Figure 4 shows the estimated mean τ and standard deviation σ of persistence duration plotted as a function of the distance Δx for each of the four subjects. Of the 28 estimated normal distributions, only one would be rejected by χ^2 at $P < 0.05$. As in the previous experiment, we found that over a limited range of spatial separations the mean duration of visible persistence increases with spatial separation. In addition, we found that for three of the four subjects (EW, JF, JG), the mean duration of visible
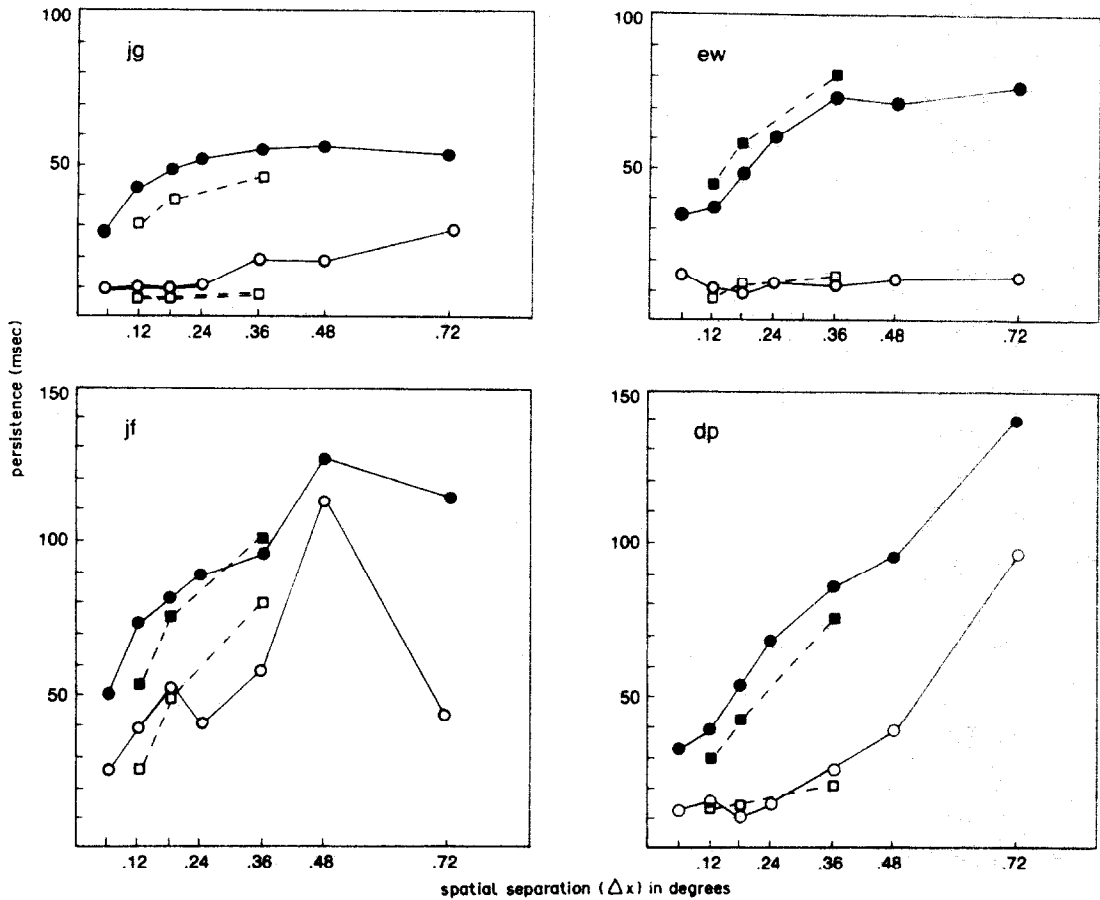
Fig. 4. Estimates of persistence duration plotted as a function of the distance separating successive sitmuli for each of the four subjects. Solid circles represent estimates of the mean persistence duration and open circles represent the standard deviation of mean persistence durations. Corresponding estimates of visible persistence derived from Expt 1 are plotted as squares.

persistence approaches a maximum (asymptote) value at the larger spatial separations. The fact that the duration of visible persistence approaches a maximum value at large spatial separations suggests that the mechanism by which the visual system modulates the duration of visible persistence operates over small spatial separations.

Figure 4 also shows that the variability in the duration of visible persistence increases with spatial separation for three of the four subjects (DP, JF, JG) and that the variability is greater at large spatial separations. Most theories of persistence would predict a correlation of τ and σ. For example, if the slope of the decaying visible persistence were to decrease over time, any variability in the threshold criteria for visibility would have greater effects at longer persistence durations. The variability in the persistence estimates for large separations is substantial, however, particularly for subjects JF and DP. This result reduces our confidence

in the persistence estimates for large spatial separations.

Finally, Fig. 4 shows the mean and standard deviation of persistence duration estimated from the results obtained in Expt 1. The estimates obtained from Expt 1 are based on stimulus conditions in which the number of successive stimuli, N, varied. The estimates obtained from expt 2 are based on stimulus conditions with constant N. Despite these differences, the mean persistence durations measured in the two experiments fall within the variability in persistence duration for each condition of spatial separation.

EXPERIMENT 3

In the previous experiments, we were able to estimate the mean τ and the variability σ of the duration of visible persistence of a briefly presented visual stimulus as a function of the distance, Δx, separating that stimulus from

other stimuli that occur later in time. We found that the estimated persistence duration τ increases with Δx, and, for 3 of 4 subjects, so does σ. We interpret the average duration τ as the time during which the response to a stimulus remains above a fixed threshold. In the following sections of this paper we examine the implications of the empirical findings in terms of more formal models. To simplify our analysis, we consider only expected values and, for the time being, we ignore variability.

The results discussed thus far may be interpreted in terms of two types of models. In one type of model the shape of the actual temporal response depends on nearby stimuli. For example, the presence of an adjacent stimulus may increase the rate of decay of the response (see Fig. 5a). In a simple exponential system this can be interpreted as a reduction in time constant. We will call this type of model the *rate of decay* model. In the second type of model, the shape of the temporal response may be invariant, only its amplitude is reduced by the presence of adjacent stimuli (see Fig. 5b). We will refer to this type of model as the *gain* model. The rate
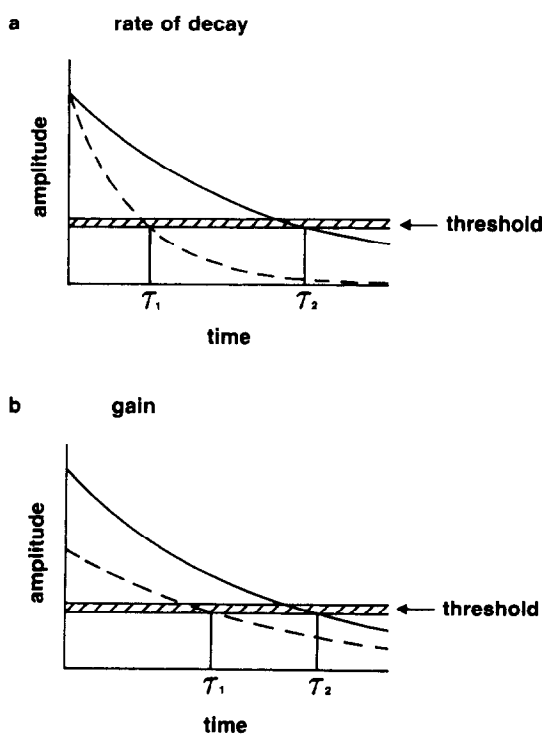

Fig. 5. Hypothetical mechanisms for modulating the duration of visible persistence. (a) The rate of decay model assumes that the presence of an adjacent stimulus increases the rate of decay, and therefore the shape, of the temporal response. (b) The gain model assumes that the shape of the temporal response is invariant, only its amplitude is reduced by the presence of adjacent stimuli.

of decay model places no constraint on the shape of the temporal response which can vary with the presence of adjacent stimuli. The gain model constrains the shape of the temporal response to be invariant and, therefore, separable from the influence of adjacent stimuli. In the sections that follow we explore the extent to which a gain model can account for the influence of adjacent stimuli on the duration of visible persistence. We first consider a more formal model of subjects' performance and then describe an experiment to address this issue empirically.

Let us denote the visibility at time t due to a stimulus with intensity l presented at time $t = 0$, $v_{\Delta x}(l, t)$. As before, Δx represents the spatial separation of adjacent stimuli. For simplicity we assume that v is monotonically decreasing (decaying) in time and monotonically increasing with luminance. The value of visibility, v, is used by the subjects to make a decision about the presence of a visible stimulus at each location.

An implicit assumption underlying our data analysis thus far is that the stimulus is visible whenever v was large enough to exceed a fixed threshold c. The estimation of the visible persistence from the results of Expts 1 and 2 amounted to estimating τ_c such that

$$v_{\Delta x}(l, \tau_c) = c. \qquad (2)$$

The estimate of mean persistence duration τ_c, or simply τ, as a function of Δx and Δt for a constant value of luminance l was justified to the extent the criterion c is independent of Δx and Δt, i.e. that the stimulus is visible whenever the visibility function v is greater than a fixed threshold value, c, and that c is constant for all Δx and Δt.

The gain type of model is based on the idea that the distance separating successive stimuli affects only the *amplitude* of the underlying temporal response, v. The amplitude of the response is likely to depend on the stimulus luminance as well. Therefore, in order to develop a gain type of model, it is necessary to separate the effects of luminance and the effects of spatial separation on the temporal response, v. To do this, we first examine the effects of luminance on estimates of persistence duration.

Method

An experiment to test the effects of luminance was performed. The method, apparatus, pro-
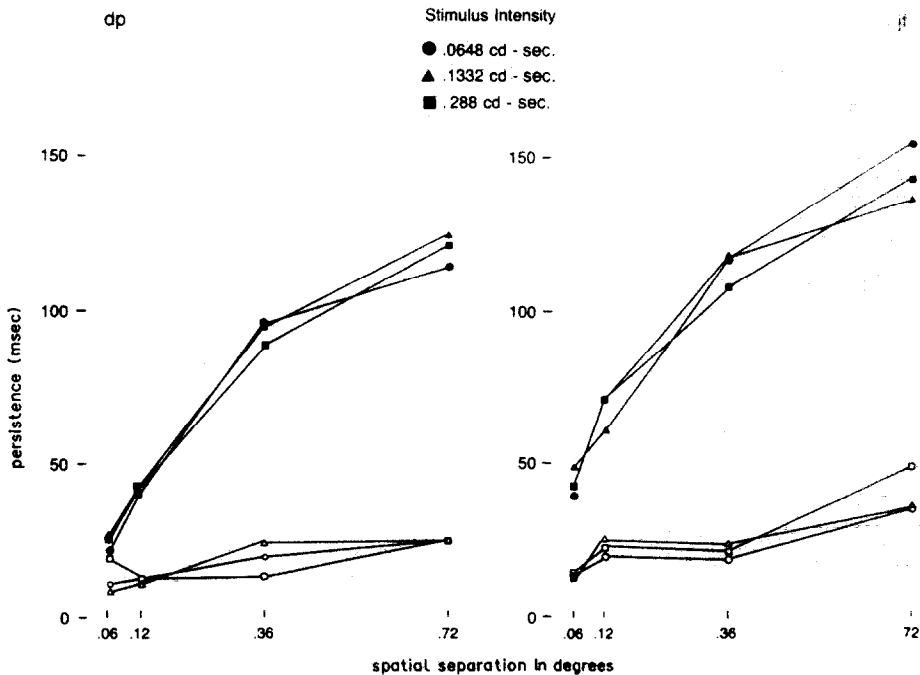
Fig. 6. Estimated mean (solid symbols) and standard deviation (open symbols) of persistence duration plotted as a function of the distance separating successive stimuli with stimulus intensity as a parameter. Stimulus intensity is specified as a fraction of the reference intensity (see *Stimuli* for Expt 1).

cedure and paradigm were identical to that of Expts 1 and 2 except that, in any given trial, the luminance of the briefly presented line was 0.72, 1.48 or 3.2 times the reference intensity (see *Stimuli* in Expt 1) and the spatial separation of successively presented lines was 0.06, 0.12, 0.36 and 0.72 deg visual angle. The distance between the centers of adjacent pixels was 0.0193 cm in both the vertical and horizontal direction.

Results

In Fig. 6, the estimated means and standard deviations in visible persistence are plotted as a function of spatial separation with stimulus luminance as a parameter for the two subjects, JF and DP. Figure 6 shows that there were no systematic effects due to stimulus luminance. Differences in the mean duration of visible persistence due to stimulus luminance are small and inconsistent and can be explained by the variability of persistence duration: for each condition of spatial separation, the mean duration of visible persistence estimated for a stimulus of a given luminance value falls within the standard deviation of the persistence durations estimated for stimuli presented in any of the three luminance values. The results of this experiment can be described very simply: the persistence estimates are invariant with respect to 1:4 luminance changes.

Discussion

The visibility criterion depends on peak visibility. The goal of the following discussion is to examine how well the data can be accounted for by a model that assumes that the visibility of a briefly presented line can be represented as a product of three different functions depending on luminance, distance and time, respectively. We begin by noting that brighter flashes do not persist longer than dim flashes. This result suggests that the criterion c depends on luminance in the same manner as does the visibility v. In other words, the results are consistent with the hypothesis that criterion is a threshold defined in terms of a fixed fraction of the initial amplitude of the visual response at time $t = 0$ which is, in turn, a monotonically increasing function of the maximum luminance.

We can express the notion of a relative criterion that is determined by the brightest stimulus on a given trial formerly as follows. Let l_m be the luminance of the brightest, briefly presented stimulus line on a given trial. Another stimulus line presented with luminance l on the same trial will be visible after a delay t if:

$$v_{\Delta r}(l, t) \geqslant c(l_m); \qquad (3)$$

where c is a monotonically increasing function of the maximum luminance.

Separability of luminance and distance effects. The threshold criterion c is, as before, assumed to be independent of the spatial and temporal stimulus parameters, Δx, Δt. At the visibility threshold, the inequality (3) becomes an equality and we can divide both sides of this equation by the threshold c. The resulting ratio $v/c = 1$ is independent stimulus luminance. Consider trials where all stimuli are presented with the same luminance l. Then $l = l_m$, the ratio v/c can be used to define a new function w:

$$w_{\Delta x}(t) = \frac{v_{\Delta x}(l, t)}{c(l)}; \qquad (4)$$

which does not depend on the luminance level. We have already defined w to be independent of luminance at threshold. If we further assume that w is independent of luminance above the threshold, then the visibility v can be written as a product of two functions:

$$v_{\Delta x}(l, t) = c(l)w_{\Delta x}(t); \qquad (5)$$

where c is a monotonically increasing function of luminance, l, and w is a monotonically decreasing function of t and increasing in Δx. Thus v is a separable function of luminance and another function w that depends on time and separation. Note that the function w is independent of luminance and embodies the dependence of persistence on spatial separation Δx.

Separability of time and distance in a gain control model. With this framework at hand, we are ready to formalize the assumption underlying the gain type of model. In that model, the presence of adjacent stimuli only modulates the magnitude of the response. That is, the function w itself can be separated into a product of two functions, gain g, and temporal response h, as follows:

$$w_{\Delta x}(t) = g(\Delta x)h(t);$$

and the visibility function can be written as

$$v_{\Delta x}(l, t) = c(l)g(\Delta x)h(t). \qquad (6)$$

The separability of time, distance and luminance expressed in equation (6) predicts that a decrement in luminance, could completely compensate for a corresponding increment in separation Δx. Alternatively, a decrease in the visibility due to small spatial separation can be compensated by an increase in visibility with luminance. Experiment 4 was aimed at discovering the relationship between spatial separation and luminance. If we know how the amplitude

of the visual response changes with luminance, and we know how luminance and spatial separation trade-off in determining the duration of visible persistence, then we can derive how the amplitude of the visual response changes with spatial separation.

EXPERIMENT 4

Experiment 4 tests the extent to which the gain type of model holds and thereby yields more information on the temporal response, h. The approach is based on the measurement of a trade-off between the function of luminance, $c(l)$, and the function of separation, $g(\Delta x)$. Since neither $c(l)$ or $g(\Delta x)$ depend on Δt [i.e. they are separable from $h(t)$], we investigated the effects of luminance and spatial separation when $\Delta t = 0$.

Method

Subjects. The same four observers who participated in the previous experiments (EW, DP, JG and JF) served as subjects in this experiment as well.

Stimuli. As in Expt 2, the stimuli consisted of two sets of vertical lines presented 0.12 deg above and below a fixation point (see Fig. 2). In fact, the stimuli were equivalent to the stimuli in Expt 2 with the following exceptions. Rather than present the lines successively, the lines were presented simultaneously. In addition, the intensity of each line was varied as a function of the position of the line: across a row of vertical lines, the intensity of each line decreased exponentially with stimulus position as illustrated in Fig. 7. Let l_i be the intensity of a line in position n.
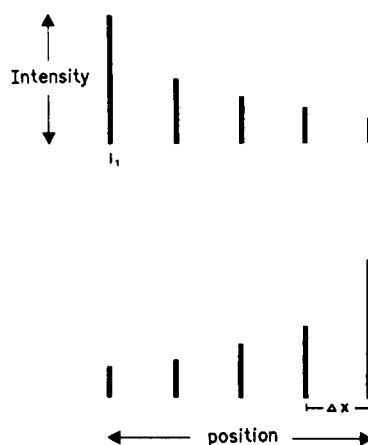

Fig. 7. The display for Expt 4: two sets of vertical lines were simultaneously presented above and below a fixation point. The height of each vertical line represents stimulus luminance which decreased exponentially with stimulus position.

The intensity of the line in the leftmost (or rightmost) position, I_i, was initialized to the reference intensity (see *Stimuli* in Expt 1). The intensity of the line in a position to the right (or left) of n, I_{n+1}, was $I_i^* \alpha$ where α is the slope of the exponential decrease. On each trial, the direction of the exponential decrease in intensity (left-to-right or right-to-left) of lines presented above the fixation point was chosen randomly; the intensity of the lines below the fixation point decreased exponentially in the opposite direction. The spatial separation δx is varied by increasing n over a range of 3–25 as in Expt 2.

Procedure. The subject initiated a trial by pressing a response key. After 600 msec, the stimuli were flashed for 1 msec. At the end of each trial, the subject pressed one of two response keys to indicate whether or not all the vertical lines above and below the fixation point were visible. Subjects were instructed to use the same criterion for visibility that they used in the previous experiments: subjects were to respond "yes" if they perceived a grating composed of all the lines above and below the fixation point and to respond "no" otherwise.

At the beginning of each session, subjects repeated 280 stimulus trials from the previous experiment. These 280 trials served to remind subjects of the visibility criterion used in previous experiments and to encourage them to use the same visibility criterion in this experiment. Subjects then viewed 3 blocks of trials, each block consisting of 160 trials. Subjects rested between blocks of trials.

Across the three blocks of trials, each condition of spatial separation Δx was presented 60 times. The 60 repetitions were presented within two interleaved staircases. The total 480 trials resulting from the product of the 7 Δx, the 60 repetitions per Δx, and the 2 staircase conditions were presented in random order.

The rate of the exponential decrease in stimulus intensity α was controlled by a modified up–down staircase. The starting value of α was 0.99. If the subject responded "yes", α was decreased by 0.01 and this new α was stored for the next presentation of this staircase. If the subject responded "no" for two repetitions of the same stimuli, α was increased by 0.01 and this new α was stored. Under the assumption that α is a normally-distributed variable, the staircase procedure converges to the α for which 71% of the time all the n lines are visible to the observer for each condition of spatial

separation, Δx. All the data were used to estimate the entire psychometric functions.

Results and discussion

Psychometric functions relating the probability of reporting that all n lines were simultaneously visible to the relative intensity of the dimmest line were calculated for each subject and each condition of spatial separation, Δx. In Fig. 8, the relative intensity of the dimmest line (expressed as the normalized ratio of the minimum and maximum line intensities) accompanying 50% response probabilities is plotted as a function of the spatial separation for each subject. As Fig. 8 shows, the relative line intensities required for all n lines to appear to be visible decreased with spatial separations up to 0.24 deg of visual angle. For larger spatial separations, the relative line intensities required to see all n lines do not vary systematically and therefore we conclude that the intensities are independent of spatial separation.

The results of Expt 4 can be interpreted in terms of the gain control model. In particular, considering the form of the visibility function v given by equation (6) we set $t = 0$ and interpret Expt 4 as finding values of the dimmest, N-th line I_N for each Δx such that:

$$c[I_N(\Delta x)]g(\Delta x)h(0) = c(I_1);   \qquad (7)$$

where I_1 is the first (brightest) line. There are three unknown functions in this equation c, g and h and our goal is to determine h. We do that in two steps. First, we use previous information on intensity scaling to assume a reasonable form for the criterion function c. We then combine the results of Expts 1, 2 and 4 in order to eliminate g.

The criterion function c represents the observers' adjustments to changes in luminance. To proceed with our analysis we need to make an additional assumption about the function $c(I)$. In particular, we assume $c(I)$ to be a power law. This assumption is consistent with at least two empirical considerations. First, the classical scaling data derived from magnitude estimation experiments (Stevens, 1957) suggests that perceived brightness is a linear function of luminance raised to a power. Second, the assumption is consistent with the luminance invariance observed in Expt 2.

Substituting I^β for c in equation (7) yields:

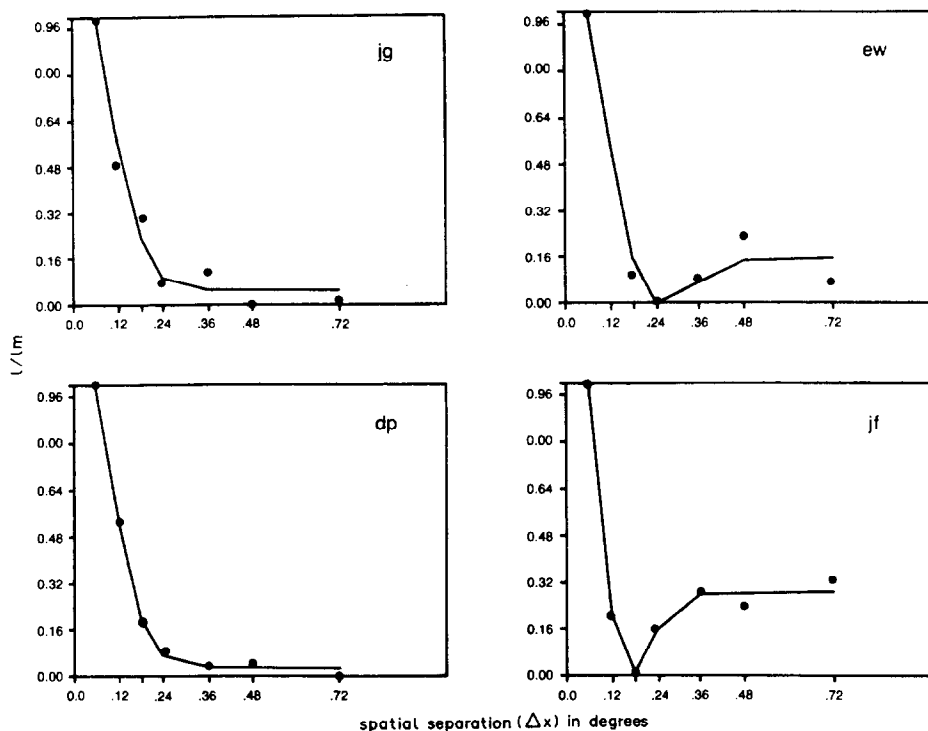$$I_N^\beta(\Delta x)g(\Delta x) = c_0 I_1^\beta.$$

Fig. 8. The relative intensity of the dimmest line (expressed as the normalized ratio of the minimum and maximum line intensities) accompanying 50% response probabilities is plotted as a function spatial separation for each subject. The solid line represents the fit between equation (10) (see text) and the data.

where c_0 is a constant [incorporating $h(0)$]. Taking logarithms of both sides yields the following equation relating Δx and l:

$$\log[g(\Delta x)] = \log(c_0) - \beta \log\left[\frac{l_N(\Delta x)}{l_1}\right]. \quad (8)$$

This equation represents the relationship between two functions of the stimulus separation $l_N(\Delta x)$ and $g(\Delta x)$. Our primary goal is to use the equation (8) to combine the results of Expt 4 with those of the earlier experiments and directly evaluate the shape of the temporal response, h. It is also possible, however, to examine whether there exist plausible gain functions g consistent with both equation (8) and the results of Expt 4. In order to find such a g we first determined a functional form for $l_N(\Delta x)$. While there are many different functions consistent with the empirical constraints on l, we selected the following spatial weighting function generated by taking the difference between two Gaussian functions:

$$\frac{l_N(\Delta x)}{l_1} = A_1\phi(\Delta x, \sigma_1) - A_2\phi(\Delta x, \sigma_2); \quad (9)$$

where $A_i > 0$ are the amplitudes, $\sigma_i > 0$ standard deviations of the positive ($i = 1$) and negative

($i = 2$) Gaussians, respectively, and where ϕ is a Gaussian density function of the form:

$$\phi(x, \sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x}{\sigma}\right)^2};$$

centered at the origin. This type of spatial weighting function seemed plausible because, given the correct parameters, it has been used to describe other spatial interactions including empirically observed receptive fields in monkey and cat retinal ganglion cells (Enroth-Cugel & Robson, 1966). The difference between two Gaussian functions has also been used to approximate psychophysically defined spatial weighting functions (e.g. Schade, 1956; Wilson & Bergen, 1979; Graham, 1980).

The best-fitting parameters to equation (9) were derived for each subject using an iterative fitting procedure (STEPIT, Chandler, 1965) that minimized the squared error between each subject's data and equation (11). The resulting fits, shown in Fig. 8, are quite reasonable. The root-mean-square error for the fits is 0.05, 0.013, 0.058 and 0.025 for subjects JG, DP, EW and JF, respectively.

Since the dependence of the luminance on Δx can be characterized as a difference of two

Gaussian functions then the resulting gain function g, shown in Fig. 8, is also a difference of Gaussians but raised to a positive power β. This function:

$$g(\Delta x) = k[A_1\phi(\Delta x, \sigma_1) - A_2\phi(\Delta x, \sigma_2)]^\beta;$$

where k is a positive constant, appears to be a reasonable reflection of the effect of spatially adjacent stimuli on persistence. According to these results, the width of the *effective field* within which one stimulus line affects the persistence of another is approx. 0.24 deg of visual angle. Since the form of the gain function appears to be reasonable we proceed to use the data from Expt 4 to derive the temporal dependency h. Note that the following derivation is independent of the form of the gain function.

Derivation of temporal dependency. Assuming that the gain model holds, the temporal waveform of the underlying visual response to a briefly presented visual stimulus is embodied in the function h. To evaluate h we need to eliminate g in equation (7). We accomplish that by substituting, in equation (7), the expression for g from equation (8). Empirically, this amounts to combining the results of Expt 4 with those of Expts 1 and 2.

To combine equations (4) and (8) we first take the logarithm of both sides of equation (7), and then solve for g with the result $\log[g(\Delta x)] = \log[b] - \log[h(t)]$. Then, substituting for $\log[g(\Delta x)]$ in equation (8) yields:

$$\log[b] - \log[h(\tau)] = \log(c_0) + \beta \log\left[\frac{l_m}{l}\right];$$

which can be simplified to:

$$\log[h(\tau)] = k + \beta \log\left[\frac{l}{l_m}\right]; \qquad (10)$$

where k is a real constant. To estimate the temporal decay function h consistent with our results can be accomplished by finding a function of τ which is linear in $\log[l/l_m]$.

For each subject and each condition of spatial separation, $\log[l/l_m]$ was estimated by the 50% threshold criteria of psychometric functions relating the probability that the subject responded "yes" (to indicate that all stimuli were visible) to the ratio of the minimum and maximum stimulus luminances, l/l_m. Figure 9 shows $\log[l/l_m]$ plotted as a function of $\log(1/\tau)$ (derived from the data collected in Expt 2) for each subject. The solid lines in Fig. 9 illustrate that the following equation provided a reasonable fit to

the data for spatial separations less than or equal to 0.24 deg of visual angle:

$$\log\left(\frac{1}{\tau}\right) = k + a \log\left[\frac{l}{l_m}\right]. \qquad (11)$$

We can therefore conclude that, to the extent that this equation is supported by the data, the gain model cannot be rejected (at least for small spatial separations) and that the decay of visible response has the general form $1/t$. This function might not be a realistic impulse response for a linear system, but it does indicate that the decay of visible persistence is slower than a simple exponential (cf. Rumelhart, 1969; Hawkins & Shulman, 1979; DiLollo, 1984).

Finally, Fig. 9 shows that for larger values of τ (corresponding to stimulus conditions in which Δx was greater than 0.24 deg of visual angle) there seems to be systematic departure from the straight line. This represents the failure of the model to capture spatial interactions over larger separations.

GENERAL DISCUSSION

Persistence is a property of any linear system with limited temporal bandwidth. Usually, the narrower the bandwidth the longer the persistence. Similarly, the more veridical the temporal response of a system is, the less persistence there is. In any sensing system, there is a trade-off between the ability to reproduce the temporal properties of a stimulus (achieved by broad temporal bandwidth and, consequently, short persistence) and the ability to detect the presence of a weak stimulus in the presence of noise (achieved by temporal summation and, consequently, long persistence). There are many situations in which the visual system sacrifices temporal bandwidth in favor of stimulus sensitivity. For example, the time constant of temporal integration is more than two times longer in the dark adapted eye than in the light adapted eye. (Sperling & Sondhi, 1968). We report an instance in which, depending on the spatio-temporal properties of the stimulus, the visual system sacrifices either temporal bandwidth or stimulus sensitivity. When the distance between successive stimuli is small, as in the case of the apparent motion of a single object, the visual system sacrifices stimulus sensitivity in favor of temporal fidelity, preserving the temporal stimulus information and reducing the smear that would otherwise be generated by moving objects (Burr, 1980). When the distance between
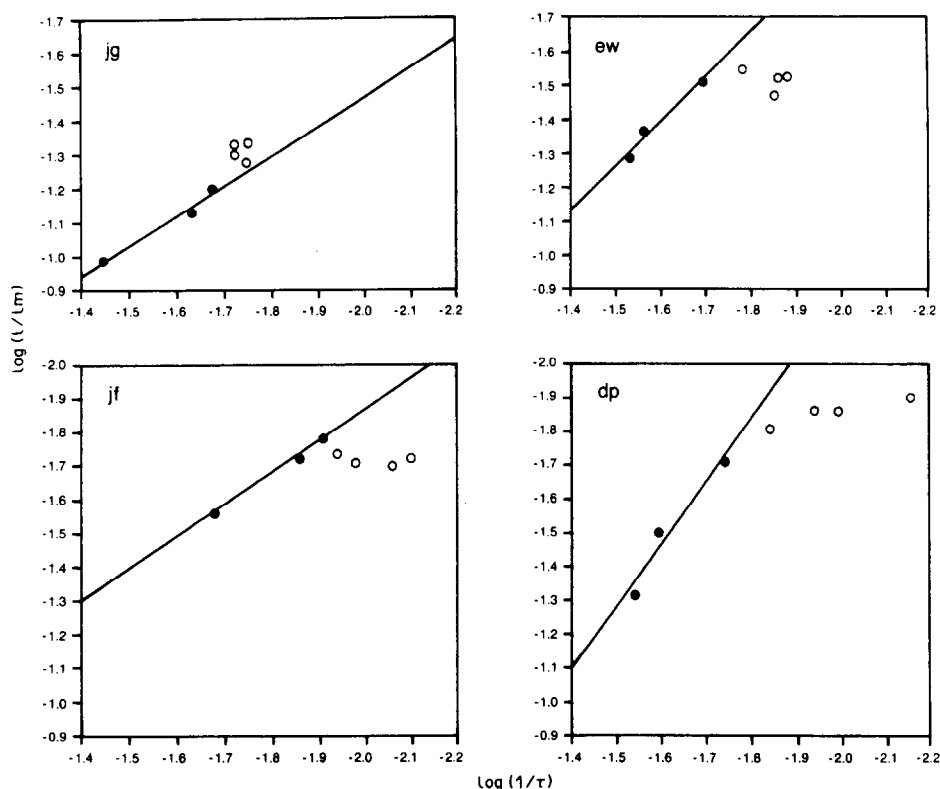
Fig. 9. For each subject, the natural logarithm of l_m/l (the ratio of maximum (l_m) and minimum (l) stimulus luminance that accompanied the 50% visibility threshold criteria in Expt 3) is plotted as a function of $1/\tau$ (the reciprocal mean duration of visible persistence estimated from the results of Expt 2). The solid lines represent the linear regression of $\ln[l/l_m]$ on $\ln(1/\tau)$ for the data corresponding to conditions in which adjacent stimuli were separating by distances less than or equal to 0.24 deg visual angle. The solid circles falling near the regression line from left to right correspond to spatial separations of 0.6, 0.12, 0.18 and 0.24 deg visual angle, respectively. The unfilled circles correspond to spatial separations greater than 0.24 deg visual angle.

successive stimuli is large, as in the case of briefly presented stationary objects, the visual system sacrifices temporal fidelity in favor of stimulus sensitivity, allowing more time to extract the spatial information necessary for object identification.

We consider a simple *gain* model as a possible mechanism for modulating the duration of visible persistence as a function of the distance separating stimuli. In this model, the shape of the underlying visual response is preserved and only its amplitude is modulated by the presence of adjacent stimuli. Our analysis does not assume any particular shape of the temporal impulse response function. We only assume that a briefly presented luminous line generates a visual response that decays over time, and that after some time the visual response reaches a threshold below which it is no longer visible. The effective duration of visible persistence corresponds to the duration that the visual response generated by the stimulus is above

threshold. In order to test the gain model, we make the further assumption that the amplitude of the visual response to briefly presented stimuli increases with stimulus luminance and that the effects of spatial separation, luminance and temporal separation on visible persistence are separable. The trade-off we observed between the effects of spatial separation and stimulus luminance on the duration of visible persistence supports the assumption of separability and the gain model. The gain model is appealing because it can be realized by mechanisms underlying shunting lateral inhibition (Sperling & Sondhi, 1968).

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