**Reply to Byrne and Hilbert**

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Byrne and Hilbert have posted a commentary entitled 'Hoffman's "Proof" of the possibility of spectrum inversion,' at this URL: http://web.mit.edu/abyrne/www/hoffmansproof.pdf. In their commentary they try to construct a simple counterexample to the Scrambling Theorem. This Theorem models functional relations between color experiences $X$ by a function, $f: X \times \ldots \times X \to W$. For instance, $f$ might be a distance metric and $W$ the nonnegative reals. Then $f(x_1, x_2)$ is the distance between colors $x_1$ and $x_2$, where smaller distances indicate greater similarity. In trying to construct a counterexample, Byrne and Hilbert take $W$, instead of $f$, to model functional relations. Doing so, they discover, leads to bizarre conclusions.

I reply as follows.

Addition and 2 are distinct. The number 2 is in the range of addition, as in 1+1=2. But elements of a function's range are not the function: 2 is not addition. A function $f: X \times \ldots \times X \to W$ is not its range $W$.

The Scrambling Theorem models functional relations by a function $f: X \times \ldots \times X \to W$. The proof of the Scrambling Theorem then follows trivially. Byrne and Hilbert conflate a function and its range—by using $W$, not $f$, to model functional relations. They discover that bizarre consequences follow. Indeed. If one conflates 2 and addition, strange consequences follow. If one conflates a distance metric and a real number, strange consequences follow. This says nothing, of course, about the Scrambling Theorem, only about the dangers of conflation. Conflation of function and range misleads Byrne and Hilbert to conclude that the Scrambling Theorem conflates epistemic and metaphysical possibility because, in part, it "makes no use of the special features of *functional* states." Well, here is one special feature of functional states that the Scrambling Theorem uses: They have the complex structure of functions; they are not unstructured points in the range of a function. If one ignores this crucial feature of functional states, as Byrne and Hilbert do, then of course problems arise.

On a separate issue, Byrne and Hilbert question the notation $b(X')$. The question is surprising. This is standard notation, e.g., for defining a continuous function: If $(X, X')$ and $(Y, Y')$ are topological spaces, then a function $b: X \to Y$ is continuous if, for all open sets $A \in Y'$, $b^{-1}(A) \in X'$. Thus, in this standard notation, $b(X') = \{b(C) \mid C \in X'\}$, where $b(C) = \{b(x) \mid x \in C\}$. Byrne and Hilbert propose that the proper definition of $b(X')$ is $\{b(x) \mid x \in X'\}$. This is an elementary mistake: $x$ does not index $X'$.

Byrne and Hilbert close by quoting Kripke (1976, 419), saying, "There is no mathematical substitute for philosophy." Yes. But one virtue of mathematics in philosophy is precision. A branch of philosophy made sufficiently precise might, we can hope, beget a science.

Kripke, S. 1976. "Is There a Problem about Substitutional Quantification?', *Truth and Meaning*, eds. G. Evans and J. McDowell, Oxford: Oxford University Press.