Deriving Physics From Conscious Agents: Transforming View-Look To Space-Time
Working Paper

Consider two conscious agents, A_1 and A_2, on a two-dimensional state space. Each point of the space, i.e., each possible state, is a conscious observer. For brevity, call each state a *view*. At each step of their dynamics, each agent observes the other and, in consequence, changes its view. For brevity, call each act of observation a *look*. Then we can say that the dynamics consists of agents engaged in a sequence of looking and changing views. The trajectories of this dynamics are thus in a *view-look* framework. We wish to find a natural transformation from view-look to space-time that will ground, and generalize, relativity and quantum theory.

We develop first a step-by-step, henceforth look-by-look, description of the dynamics of the agents. For this, consider Figure 1.

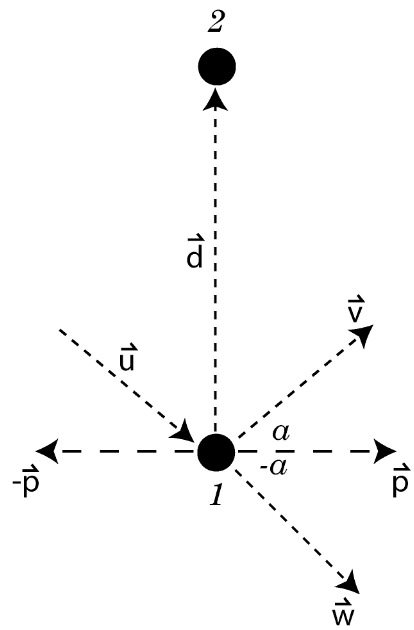

Figure 1. Parameters of agent dynamics.

The black dots represent the current views of agents A_1 and A_2. The vector difference in view from A_1 to A_2 is $\boldsymbol{d}$. When an agent looks, it learns only the direction, but not the distance, of the agent at which it looks. In consequence of what it learns, the agent changes its view. We will call this change in view a *step*. Thus, each look induces an agent to take a step. When taking a step, an agent has a fundamental choice: Engage in a relationship with the agent at which it looks, or not. This choice is represented in Figure 1 by the vectors $\boldsymbol{p}$ and $-\boldsymbol{p}$, which are orthogonal to $\boldsymbol{d}$. If A_1 takes a step, such as $\boldsymbol{v}$, whose direction lies in the hemicircle bisected by $\boldsymbol{d}$ and bounded by $\boldsymbol{p}$ and $-\boldsymbol{p}$, then A_1 *approaches* A_2. If, instead, A_1 takes a step, such as $\boldsymbol{w}$, whose direction lies in the hemicircle bisected by $-\boldsymbol{d}$ and bounded by $\boldsymbol{p}$ and $-\boldsymbol{p}$, then A_1 *leaves* A_2. If A_1 and A_2 consistently, i.e., at each look, approach each other, then their dynamics is stable:

asymptotically their differences in view are described by a probability measure. We say that A_1 and A_2 have *engaged in relationship*. If A_1 and A_2 consistently leave each other, then their dynamics is unstable: asymptotically their differences in view are not described by a probability measure. We say that A_1 and A_2 have *not engaged in relationship*. Their dynamics wanders off to the cemetery. Here we consider only the dynamics of agents engaged in relationship.

If A_1 is engaged in relationship with A_2, then there are many ways in which A_1 can approach A_2: there is an entire hemicircle of possible step directions, and an unbounded range of possible step magnitudes. We make three assumptions about how A_1 takes a step:

1. **Characteristic Angle**: A_1 has a characteristic step angle, a. This is illustrated in Figure 1. The angle is measured from $\boldsymbol{p}$ or $-\boldsymbol{p}$, and its magnitude lies in the interval $(0,\pi/2]$. This angle will also be called the *approach angle.*
2. **Characteristic Length**: A_1 has a characteristic step length, l.
3. **Consistent Direction**: At each look, A_1 steps in a manner consistent with its previous step. Two steps have *consistent direction* if the dot product of their vector representations is positive. This is illustrated in Figure 1. In accordance with our first two assumptions, A_1 could take step $\boldsymbol{v}$ (assuming $\boldsymbol{v}$ has magnitude l). However, A_1 could also take step $\boldsymbol{v}'$, where $\boldsymbol{v}'$ is the reflection of $\boldsymbol{v}$ about $\boldsymbol{d}$. The step $\boldsymbol{v}'$ makes an angle a with $-\boldsymbol{p}$, and has the same magnitude as $\boldsymbol{v}$**.** So our first two assumptions permit either step $\boldsymbol{v}$ or step $\boldsymbol{v}'$. Suppose, however, that A_1's previous step was $\boldsymbol{u}$, as illustrated in Figure 1. Then step $\boldsymbol{v}$ has a positive dot product with $\boldsymbol{u}$, and $\boldsymbol{v}'$ has a negative dot product with $\boldsymbol{u}$. Thus the only step with consistent direction is $\boldsymbol{v}$. Intuitively, the step with consistent direction is the one which least changes direction from that of the previous step.

Given these assumptions, we can speak of a *characteristic step* of an agent, viz., a step whose angle is the characteristic angle and whose length is the characteristic length. For the moment we restrict attention to dynamics involving two agents with identical characteristic steps. We later discuss dynamics involving more agents and differing characteristic steps.

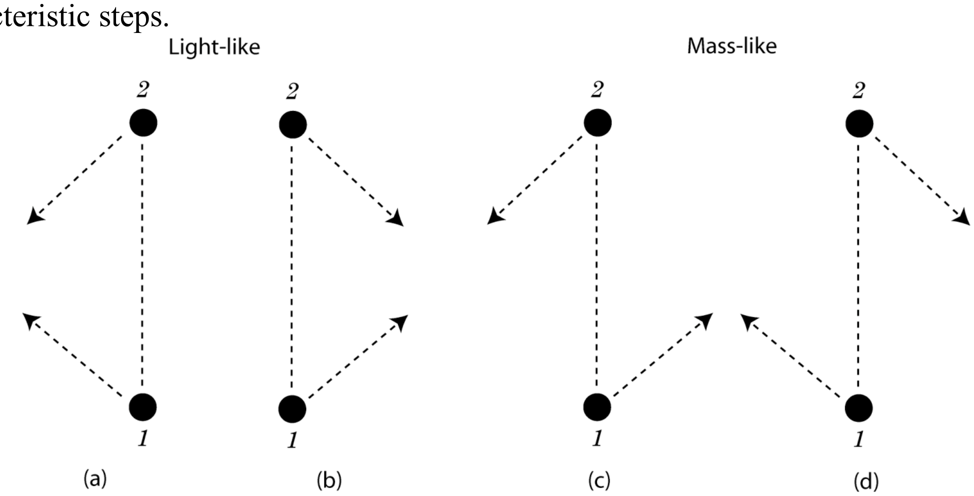

Figure 2. Four possible combinations of steps. Combinations (a) and (b) lead to boson dynamics. Combinations (c) and (d) lead to fermion dynamics.

For a dynamics of two agents there are, at each look of the dynamics, four possible combinations of steps. These are illustrated in Figure 2. In 2a and 2b, the agents both step to the same side. In 2c and 2d, the agents step to opposite sides. These different combinations lead to qualitatively different asymptotic behaviors of the dynamics.

For instance, here are 10 steps of a dynamics of type 2a:

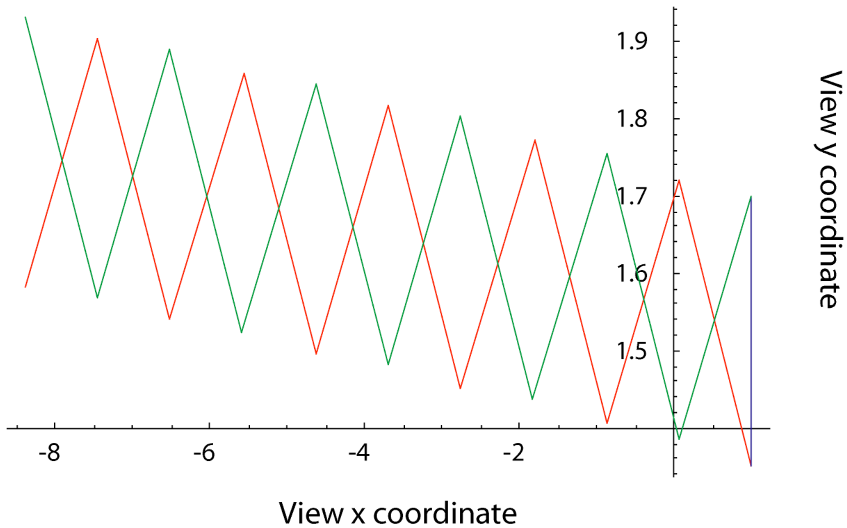

Figure 3. Boson dynamics of two agents, as in Figure 2a. The steps of agent A_1 *are in red. The steps of agent* A_2 *are in green. A blue line marks the line segment between the agents at the first look of the dynamics. The approach angle is 20 degrees. Step length is 1.*

Here are 10 steps of a dynamics of type 2b, with the initial views of the two agents identical to that in Figure 3.

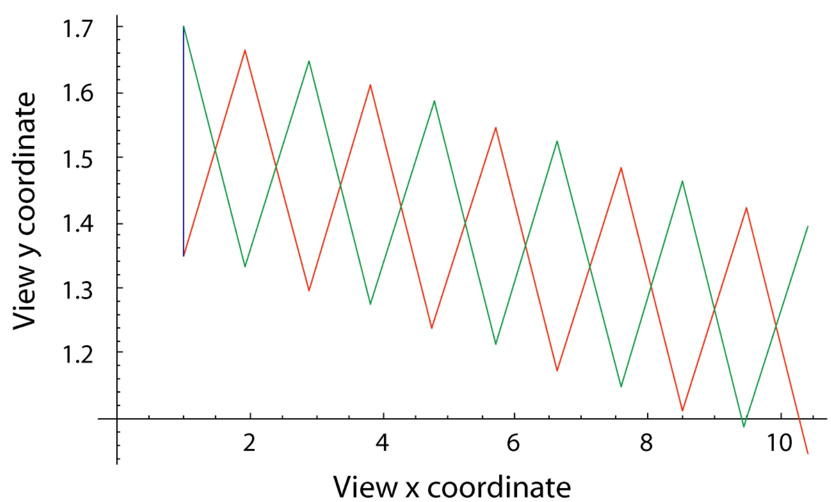

Figure 4. Boson dynamics of two agents, as in Figure 2b. The steps of agent A_1 *are in red. The steps of agent* A_2 *are in green. A blue line marks the line segment between the agents at the first look of the dynamics. The approach angle is 20 degrees. Step length is 1.*

Observe that the dynamics in Figure 3 travels in a direction opposite to that in Figure 4.

Here are 50 steps of a dynamics of type 2c, with the initial views of the two agents identical to that in Figure 3.

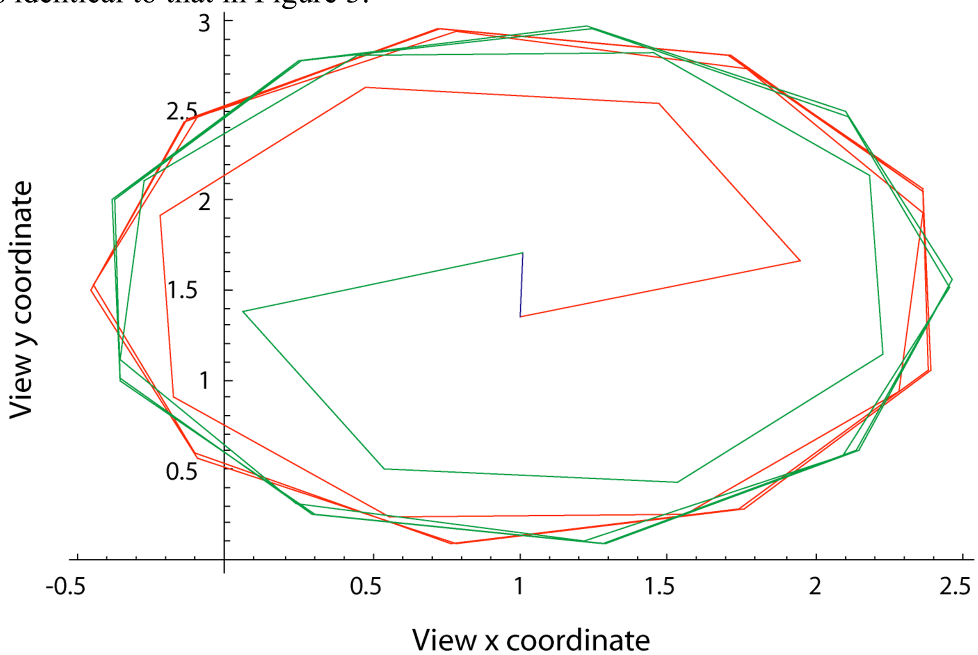

Figure 5. Fermion dynamics of two agents, as in Figure 2c. The steps of agent A_1 *are in red. The steps of agent* A_2 *are in green. A blue line marks the line segment between the agents at the first look of the dynamics. The approach angle is 20 degrees. Step length is 1.*

Here are 50 steps of a dynamics of type 2d, with the initial views of the two agents identical to that in Figure 3.

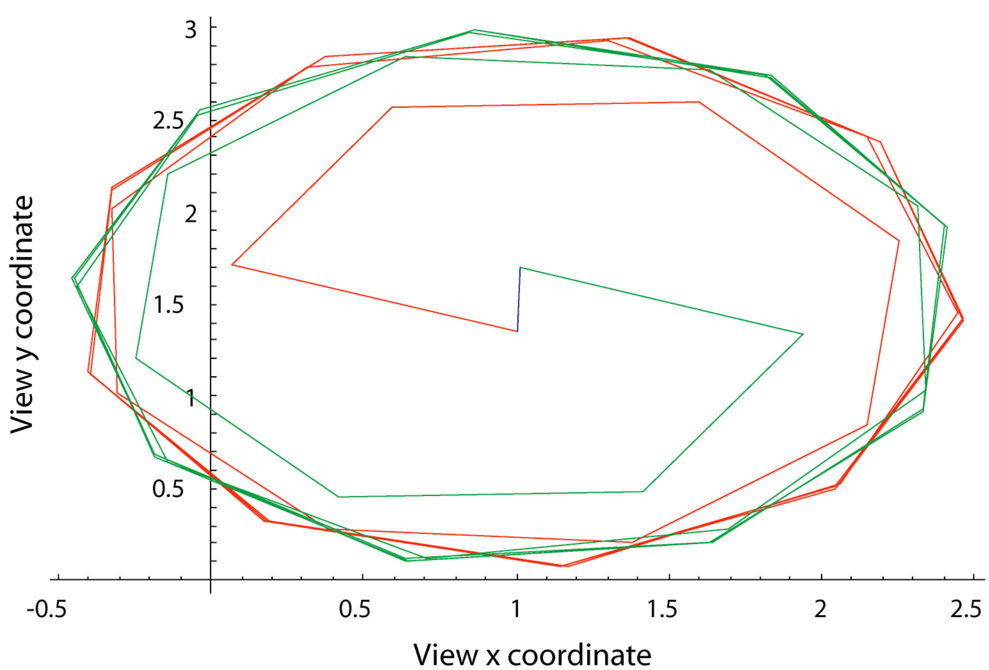

Figure 6. Fermion dynamics of two agents, as in Figure 2d. The steps of agent A_1 *are in red. The steps of agent* A_2 *are in green. A blue line marks the line segment between the agents at the first look of the dynamics. The approach angle is 20 degrees. Step length is 1.*

Observe that the dynamics in Figure 6 proceeds clockwise, whereas that in Figure 5 proceeds counterclockwise. Also observe that the center of mass of the dynamics of Figures 5 and 6 remains approximately stationary, whereas the center of mass of the dynamics of Figures 3 and 4 constantly translates. This is one reason for calling the dynamics in Figures 5 and 6 "fermion" and the dynamics in Figures 3 and 4 "boson."

If two agents engage in a boson dynamics, their asymptotic behavior depends critically on their characteristic angle. Define the *pair speed* of the agents to be the mean change in view of the centroid of the pair per step of the dynamics. Then the larger the characteristic angle the slower the pair speed. For instance, Figure 7a shows a pair of agents in boson dynamics with a characteristic angle of 20 degrees, and Figure 7b shows a pair with an angle of 60 degrees. Even though they have identical initial views, the two pairs travel different distances through view space in 10 steps. The pair in 7b has a larger angle and has a slower pair speed than the pair in 7a. The pair speed of a boson pair is proportional to the cosine of their characteristic angle.

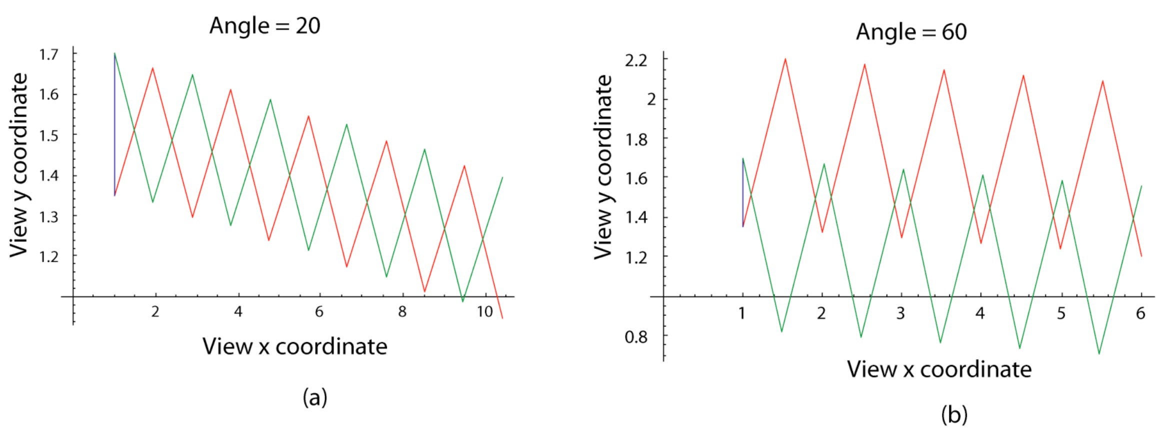

Figure 7. Two boson pairs of agents with differing characteristic angles. The rate of travel of a pair is proportional to the cosine of its approach angle. Step length is 1.

If two agents engage in a fermion dynamics, again their asymptotic behavior depends critically on their characteristic angle. In particular, the smaller the characteristic angle the greater the asymptotic distance between the agents, and the greater the number of steps required to complete one cycle around each other. For instance, Figure 8a shows a pair of agents in fermion dynamics with a characteristic angle of 20 degrees, and Figure 8b shows a pair with an angle of 10 degrees. Even though they have identical initial views, the two pairs approach different asymptotic distances in 50 steps. The pair in 8b has a smaller angle and a larger asymptotic distance than the pair in 8a. Moreover, more steps are required for the pair in 8b to complete one cycle than for the pair in 8a.

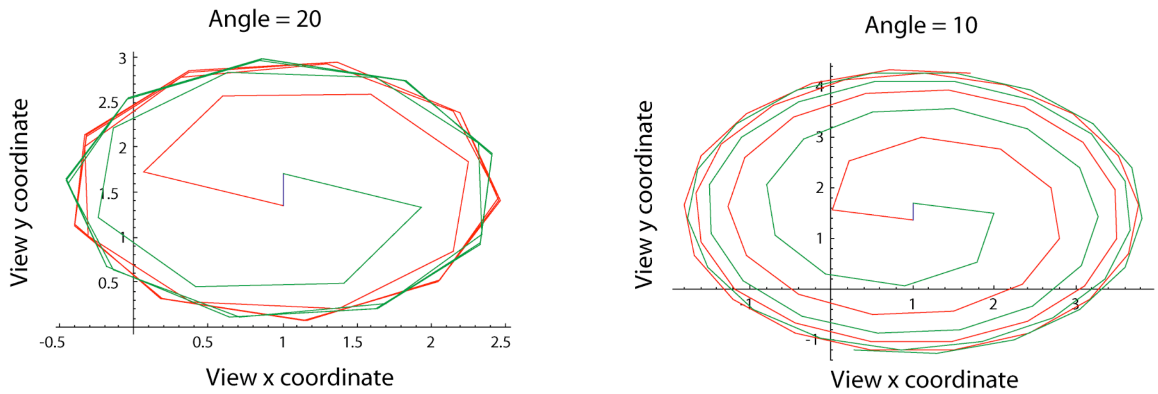

Figure 8. Two fermion pairs of agents with differing approach angles. Step length is 1.

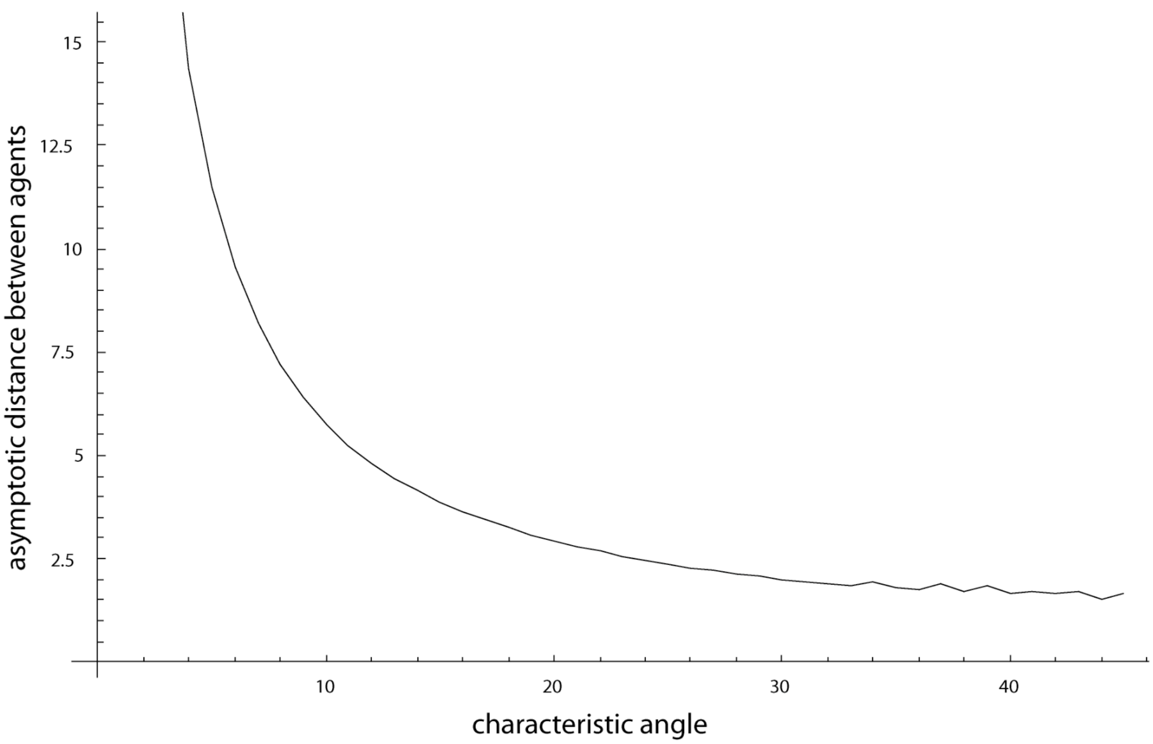

Figure 9. The asymptotic distance between two agents as a function of characteristic angle for agents in fermion dynamics. Step length is 1. For a step length d*, this function is simply multiplied by* d.

Figure 10 illustrates an approximation to the asymptotic distance between two agents having characteristic angle a and characteristic length d. The approximation is the diameter of the circle that circumscribes the polygon of agent steps. As illustrated in Figure 10, the center point of this circle is the intersection of the line $x = 0$ with the line $y - ½\, d\, sin(a) = tan(a + ½\, \pi)(x - ½\, d\, cos(a))$. Solving these two equations yields the center point $(0, ½\, d\, csc(a))$, where $csc(a) = 1/sin(a)$. Thus the diameter is $d\, csc(a)$. The area, A, of the polygon of agent steps can be approximated by the area of this circumscribed circle, viz., $A \cong \frac{1}{4} \pi d^2 csc^2(a)$. The approximation is good for small characteristic angles a. For larger angles, the approximation is poor since the polygon of agent steps has few sides. (Only if 360 is a multiple of $2a$ does the sequence of an agent's steps form a regular polygon, and have a circumscribed circle. The approximation described here can nevertheless be applied to all small values of a, since the circumcircle of a nonregular polygon of agent steps has approximately the same radius as the circumscribed circle of the regular polygon having the same number of agent steps. In the nonregular case, the polygon precesses within the circumcircle as the agents cycle.)

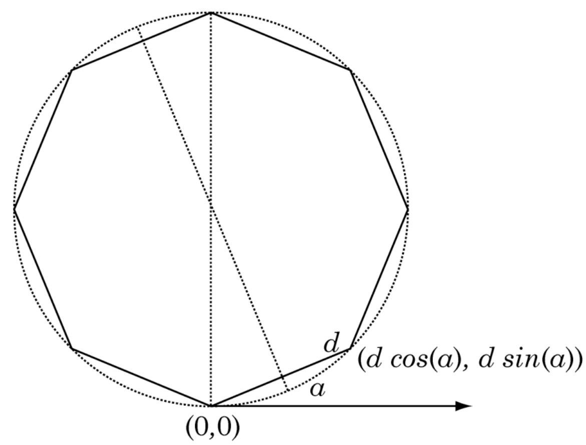

Figure 10. Approximation of asymptotic distance between agents. The approximation is the diameter of the circle that circumscribes the polygon of agent steps.

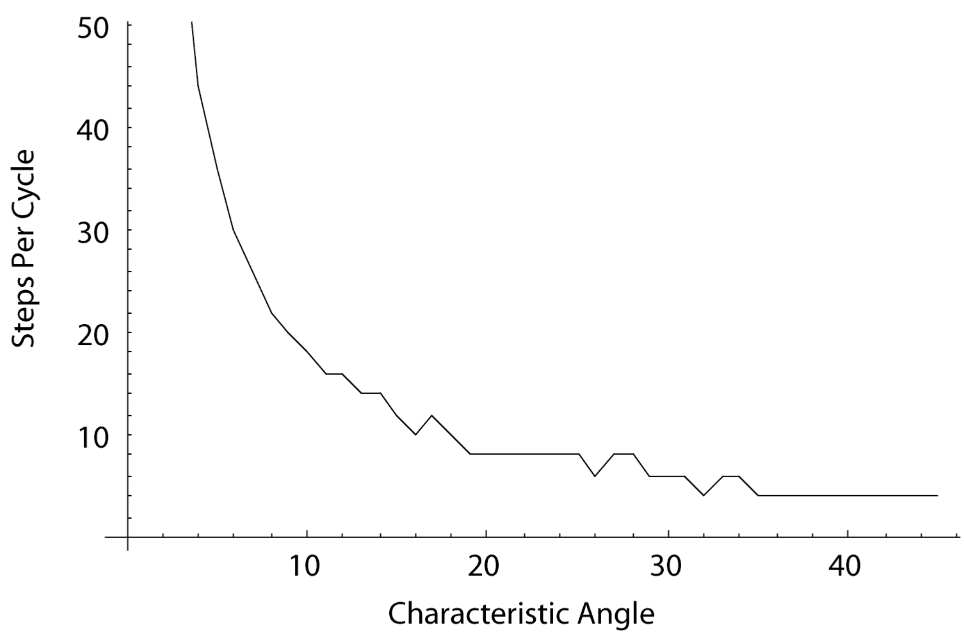

Figure 11. The number of steps per cycle as a function of characteristic angle for agents in fermion dynamics. Step length is 1.

Conscious realism claims that all that exists are conscious agents and their conscious experiences. Physical objects and space-time itself are simply species-specific user interfaces that *homo sapiens* uses to the realm of conscious agents. To substantiate this claim, we need to show how to translate view-look space and its agent dynamics into physical space-time and physical entities. The following principles guide our translation.

Principles For Constructing Physics

1. **Physical Properties**: Physics represents asymptotic properties of stable dynamical systems of conscious agents.
2. **Physical Space-Time**: The view-look pair speed of conscious agents in boson dynamics is always represented as the same velocity in space-time, viz., the velocity of light, regardless of the characteristic angle of the pair of agents.

Consider a photon. According to principle 1 for constructing physics, a photon represents asymptotic properties of a stable dynamical system of conscious agents. In particular, a photon represents asymptotic properties of a pair of conscious agents engaged in boson dynamics. According to our principle 2, a photon always travels at the speed of light, regardless of the characteristic angle of the conscious agents it represents. However, the different possible characteristic angles of the conscious agents do result in an asymptotically stable property of the pair, which therefore can be represented in the photon, viz., as the frequency of the photon. The larger the characteristic angle, a, of the conscious agents, the higher the frequency of the photon. Specifically, the frequency, ν, of the photon is $\nu = sec(a)$. From the equation $E = h\nu$, we see that the energy of a photon is determined entirely by the characteristic angle of the conscious agents.

Consider an electron. Again according to principle 1 for constructing physics, an electron represents asymptotic properties of a stable dynamical system of conscious agents. In particular, an electron represents asymptotic properties of conscious agents engaged in fermion dynamics. The different possible characteristic angles of the conscious agents do result in an asymptotically stable property, which can therefore be represented in the electron. This stable property of the dynamics is, we have seen, the asymptotic distance between the agents, the area of the polygon traced by the steps of the agents, and the concomitant number of steps per cycle. This stable property is represented in the electron as its rest mass. Since the electron has a fixed rest mass, there is a single characteristic angle of the corresponding agents. The asymptotic dynamics of the agents can cycle clockwise or counterclockwise. This is represented in the electron as spin.

As the characteristic angle approaches 90, fermion and boson dynamics in view-look space become more similar. At 90 degrees they are identical: Steps back and forth on a line. We have a supersymmetry between fermion and boson dynamics. As the characteristic angle of fermion dynamics approaches 90 in view-look space, the rest mass in the space-time representation approaches infinity, since the rest mass is inversely related to the area of the view-look dynamics. Similarly, as the characteristic angle of boson dynamics approaches 90 in view-look space, the energy of the light in the space-time representation approaches infinity, since the frequency approaches infinity. At 90, Principle 2 for constructing physics no longer allows one to construct space-time: we only get a black hole.

Since each agent maintains, in its interaction, a characteristic angle, length, and consistent direction, it follows that certain properties of their stable interactions will be conserved. These will be transformed from view-look to space-time as conservation of energy, momentum, angular momentum, and so on.

It is straightforward to extend agent dynamics from two dimensions to three. This is illustrated in Figure 12. The agents still look at each other and learn direction but not distance. That is, if agents 1 and 2 are at views $\boldsymbol{q}_1$ and $\boldsymbol{q}_2$, then all agent 1 learns about the view of agent 2 is the unit vector $\boldsymbol{d} = (\boldsymbol{q}_2 - \boldsymbol{q}_1) / |\boldsymbol{q}_2 - \boldsymbol{q}_1|$, which can be considered a point on the unit sphere of possible directions. The vector $\boldsymbol{d}$ is called a *partner vector*. Each agent still has a characteristic step length, l, and step angle a. The characteristic step angle defines a cone of possible step directions about its partner vector. This cone is indicated by the dashed circle in Figure 12. The agent chooses a unique step direction from this cone as follows: the step direction (1) is coplanar with its previous step vector, $\boldsymbol{v}$, and the partner vector, $\boldsymbol{d}$, to the other agent and (2) has positive dot product with the previous step vector. Condition (2) is the consistent direction condition. The resulting new step of length l and angle a is labeled $\boldsymbol{w}$ in Figure 12.

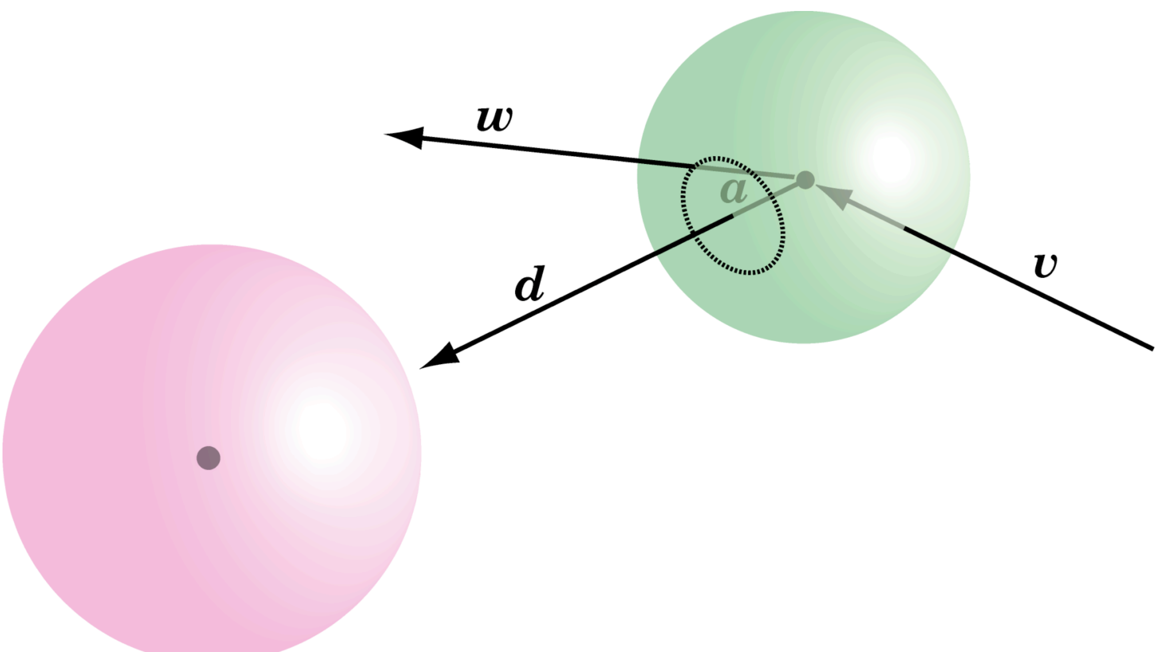

Figure 12. The geometry of agent steps in three dimensions.

The plane containing $\boldsymbol{w}$, $\boldsymbol{v}$, and $\boldsymbol{d}$ is the *action plane* for the agent at that step of the dynamics. The action plane is oriented with orientation $\boldsymbol{o} = (\boldsymbol{v} \times \boldsymbol{d}) / |\boldsymbol{v} \times \boldsymbol{d}|$. If the action planes of two agents in dynamics are identical, then their dynamics reduces to the two-dimensional case. If the action planes are not identical, then the trajectories of the agents have both curvature and torsion, with the magnitude of the torsion varying with the magnitude of the misalignment of the action planes. For boson dynamics, agents with identical action planes yield plane-polarized light; otherwise they yield elliptically- or circularly-polarized light. Figure 13 illustrates boson dynamics with misaligned action planes. The two frames shown are a stereo pair that can be fused to see the helical trajectories of the agents.

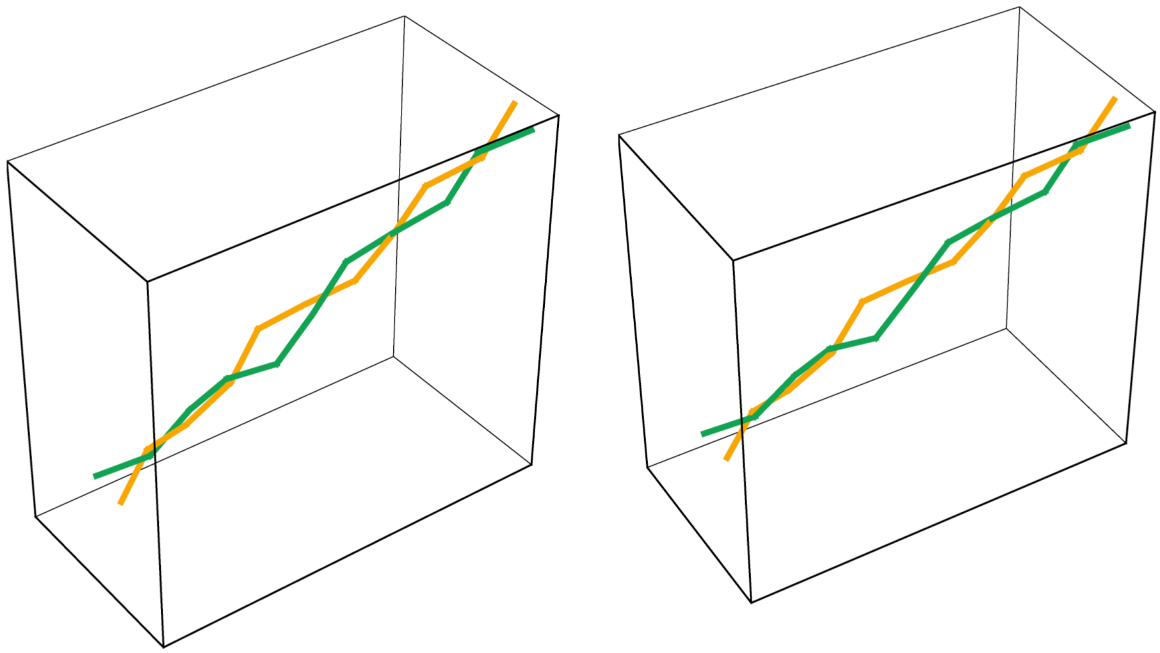

Figure 13. Boson dynamics of agents with misaligned action planes. Their trajectories are helical. This yields elliptically- or circularly-polarized light dynamics.

For fermion dynamics, misaligned action planes lead to helical trajectories, as illustrated in stereo pair of Figure 14.

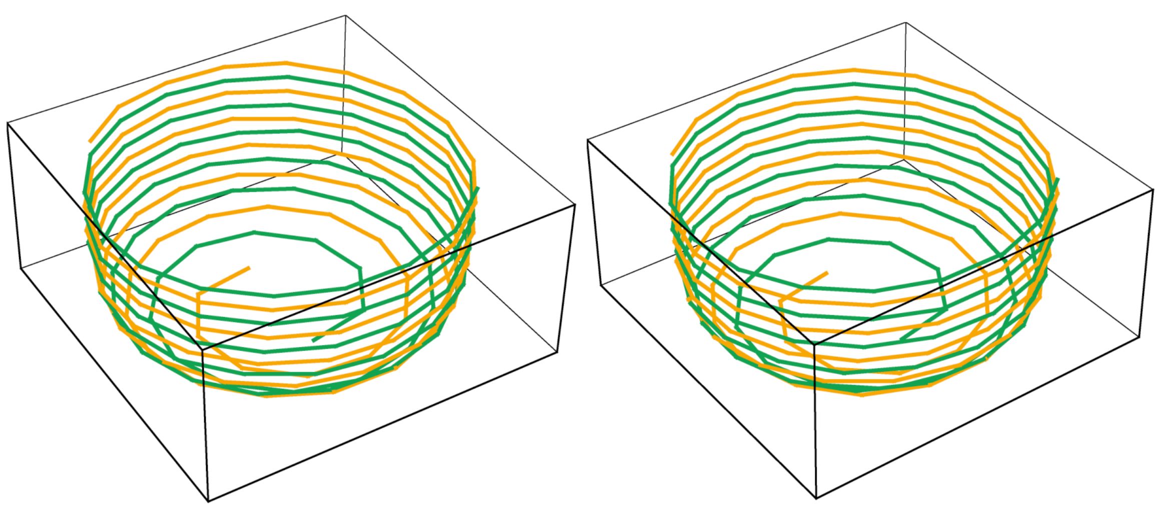

Figure 14. Fermion dynamics of agents with misaligned action planes. Their trajectories are also helical. This yields a nonzero momentum vector.

A top view of this dynamics, in Figure 15, shows that the agents still have an asymptotic area covered by their dynamics in view-look space, and therefore a mass in its space-time representation. The larger the area, the smaller the mass. Their dynamics defines an *action cylinder* in view-look space. The area of the base of this cylinder yields their particle mass in space-time, and translation along the major axis of the cylinder yields their particle velocity in space-time. Thus the action cylinder in view-look space defines the particle momentum in space-time. For agents with characteristic step of length l and angle a, the velocity v of translation of their dynamics along the cylinder axis in view-look space gets translated into a particle velocity v' in the space-time representation by a formula we develop later.

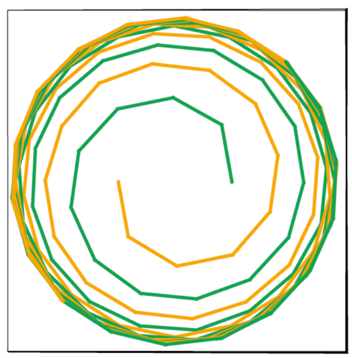

Figure 15. A top view of the fermion dynamics of Figure 14.

Earlier we discussed two principles for constructing physics from the dynamics of conscious agents on view-look space. The first principle states that “Physics represents asymptotic properties of stable dynamical systems of conscious agents.” In particular, each particle described in physics is a representation of asymptotic properties of stable dynamical systems of conscious agents. One way particles are described in physics is by their symmetry groups, such as SU(3), SU(2), and U(1). It is required, then, for us to derive these symmetry groups by classifying the symmetries of asymptotic properties of stable systems of conscious agents.

To this end, we consider two agents with a fixed, but unknown, approach angle a. Referring again to Figure 12, we first observe that the stabilities of the dynamics of two agents are independent of the direction of the difference vector between the agents’ positions in view-look space. Since this direction can lie anywhere on the unit sphere, there is an SO(3) component to the symmetries of their stabilities. Since the distance between the agents does not alter the stabilities, there is also an $\boldsymbol{R}^1$ component to the symmetries of their stabilities.

At any step of the dynamics, the jump vectors (i.e., step vectors) of the two agents completely determine the future stability of their dynamics. Each jump vector can, in principle, have any direction on the unit sphere. Thus we can think of the possible directions of jump vectors of the agents as two spheres, the *jump spheres,* attached to antipodal points of the unit sphere of possible directions between the two agents.

The cross product of a jump vector and a partner direction vector defines, as we have previously discussed a vector, $\boldsymbol{o}$, orthogonal to the *action plane*. The action plane associated to vector $-\boldsymbol{o}$ is the same plane as associated to $\boldsymbol{o}$, but with opposite orientation. The action plane is the only aspect of the jump vector that affects the stabilities of the dynamics. As shown in Figure 16, the possible action planes correspond to a great circle on the sphere of possible jump directions, orthogonal to the direction vector between the two agents. This great circle is the ***o**-circle*.

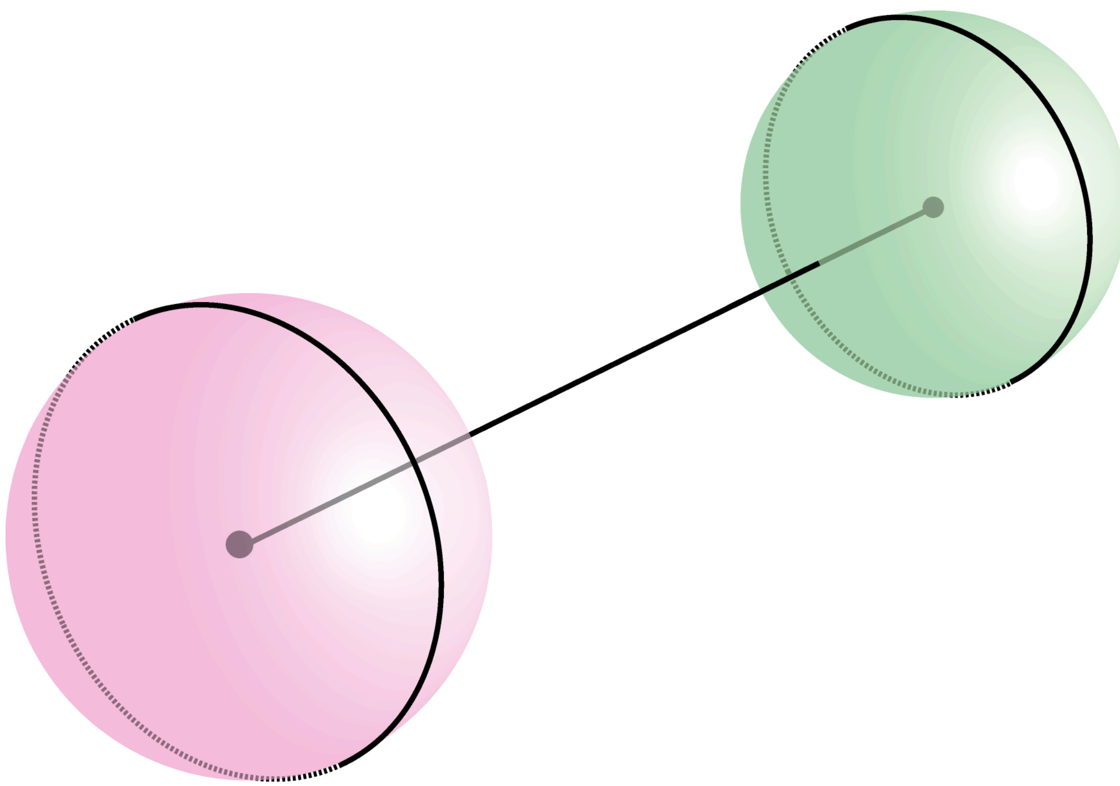

*Figure 16. The jump spheres and **o**-circles of two agents.*

The two o-circles of the two agents classify the stabilities of their dynamics. The different stabilities then correspond to *two trips* around the unit circle of differences in action planes between the two agents, trips that we can parameterize by an angle θ, that we call the *action angle*. This is illustrated in Figure 17. For $-\pi < \theta < \pi$, the dynamics is fermion. For $\pi < \theta < 3\pi$, the dynamics is boson. If $|\theta| = \pi$, the distinction between fermion and boson dynamics disappears. For boson dynamics, if $\theta = 2\pi$, then the boson dynamics is planar; otherwise it is a discrete approximation to a double helix, one helix for each agent. For fermion dynamics, if $\theta = 0$, then the fermion dynamics is planar; otherwise it is also a discrete approximation to a double helix.

Fermion dynamics for $\theta = +\varphi$, with $|\varphi| < \pi$, is identical to fermion dynamics for $\theta = -\varphi$ except that it spins in the opposite orientation and translates in the opposite direction. The translation rate increases with increasing magnitude of φ. Boson dynamics for $\theta = 2\pi + \varphi$, with $|\varphi| < \pi$, is identical to boson dynamics for $\theta = 2\pi - \varphi$ except that it spins in the opposite orientation; the translation direction is not opposite, but differs by angle φ, so that it only becomes opposite when $\varphi = \pi$.

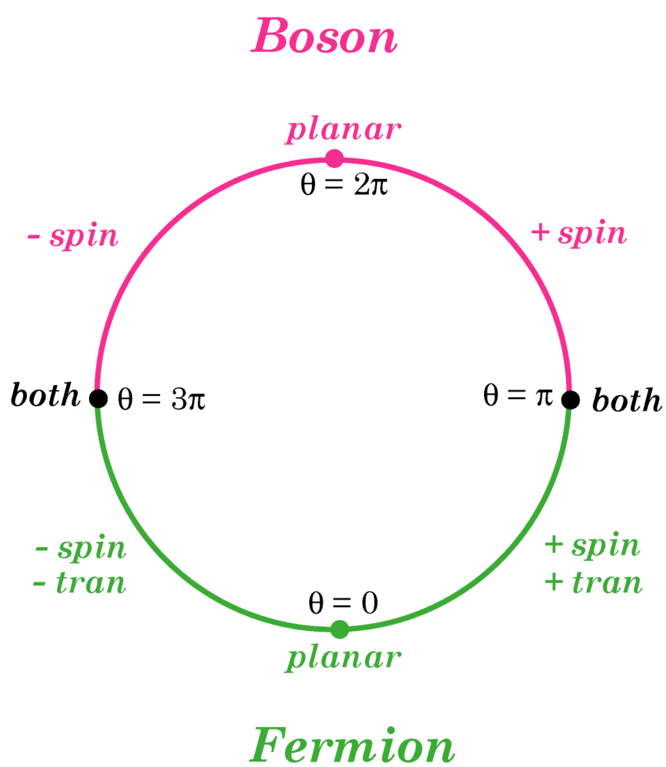

Figure 17. All possible stabilities of a pair of agents, classified by the action angle θ .

The stabilities of dynamics for an isolated pair of agents are determined by the approach angle a and action angle θ of the agents. These stabilities are captured by the helical trajectory of each agent, called the *agent helix*. For small a, arbitrary θ, and step length l, the helical trajectories are given by

$$x = r\cos(t),$$
$$y = r\sin(t),$$
$$z = ct,$$

where

$$r = l\cos(\theta/2)\csc(a),$$
$$c = l\sin(\theta/2).$$

As is standard, the curvature κ and torsion τ are given by the formulas

$$\kappa = r / (r^2 + c^2),$$
$$\tau = c / (r^2 + c^2).$$

Thus the intrinsic equations for the agent helix, $h(a, l, \theta)$, are

$$\kappa(s) = \kappa = l^{-1}\,(cos(\theta/2)\,csc(a) + sin(a)\,sin(\theta/2)\,tan(\theta/2))^{-1}$$
$$\tau(s) = \tau = l^{-1}\,(cos(\theta/2)\,cot(\theta/2)\,csc^2(a) + sin(\theta/2))^{-1}$$

This representation of the agent helix is invariant under arbitrary translations and rotations.

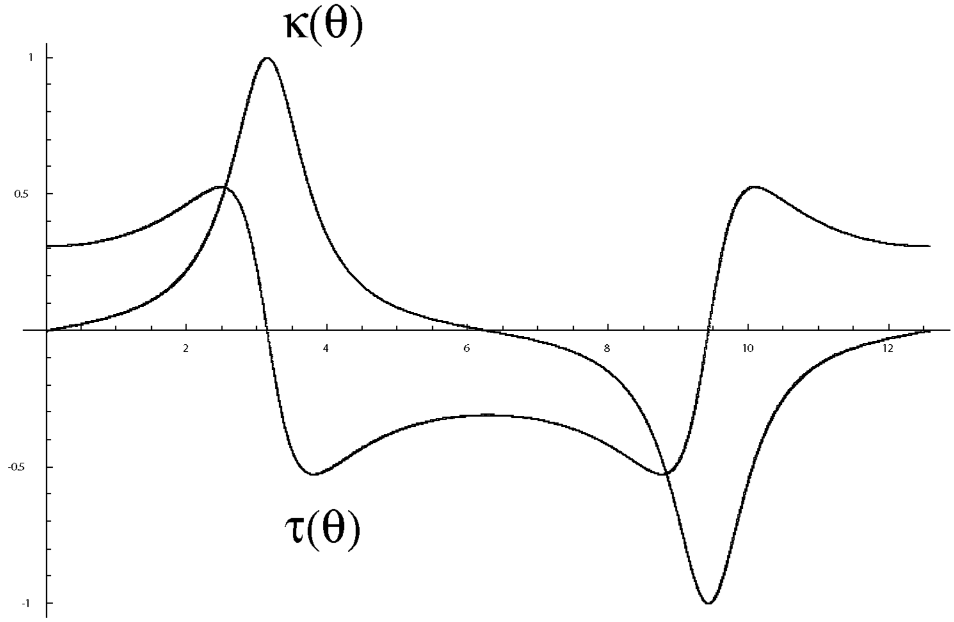

Figure. Plots of κ *and* τ *as the action angle* θ *and goes from* 0 *to* 4π. *The approach angle is* $\pi/10$ *and step length is 1.*

The vertical separation between loops of the helix is $2\pi c$. Figure 18 shows a plot of the diameter of the helix as a function of the action angle, with a fixed approach angle of 10 degrees.

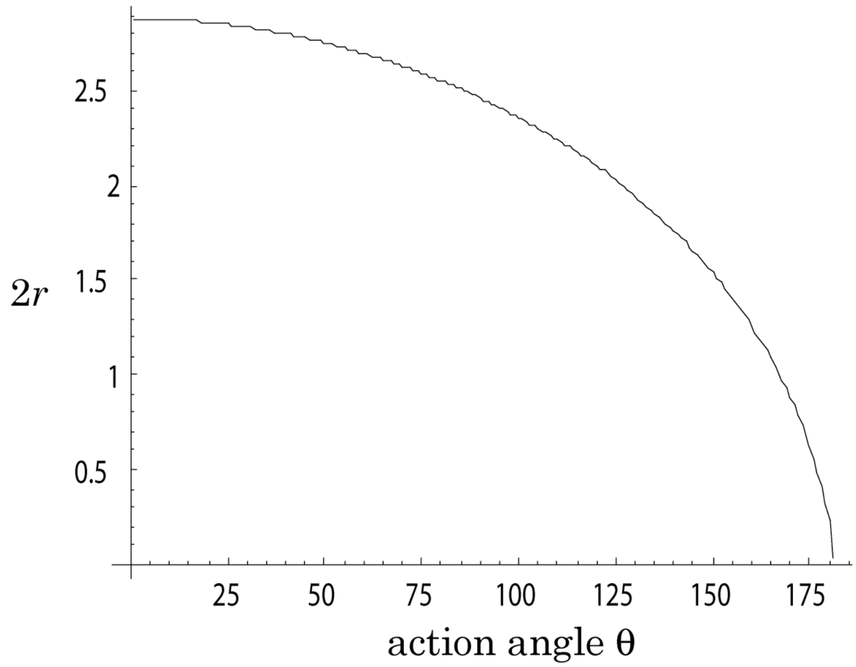

Figure 18. The diameter of the agent helix as a function of the action angle, with a fixed approach angle of 10 degrees.

Boson and fermion dynamics can be simply distinguished. If the dot product of jump vectors of the agents is positive then their dynamics is boson, if it is negative then their dynamics is fermion, and if it is zero it is both. There is no limit to the number of agents that can simultaneously engage in a stable boson dynamics, since there is no limit to how many agents can step in the same direction in a stable dynamics. However, only two agents can engage in a stable fermion dynamics, because only two agents can step on opposite directions: Add more agents and they are no longer all stepping in opposite directions; this is the origin of Pauli's exclusion principle and the difference between Bose-Einstein and Fermi-Dirac statistics. When the dot product of the jump vectors is 0, the energy and temperature are infinite, and the statistics become Maxwell-Boltzmann.

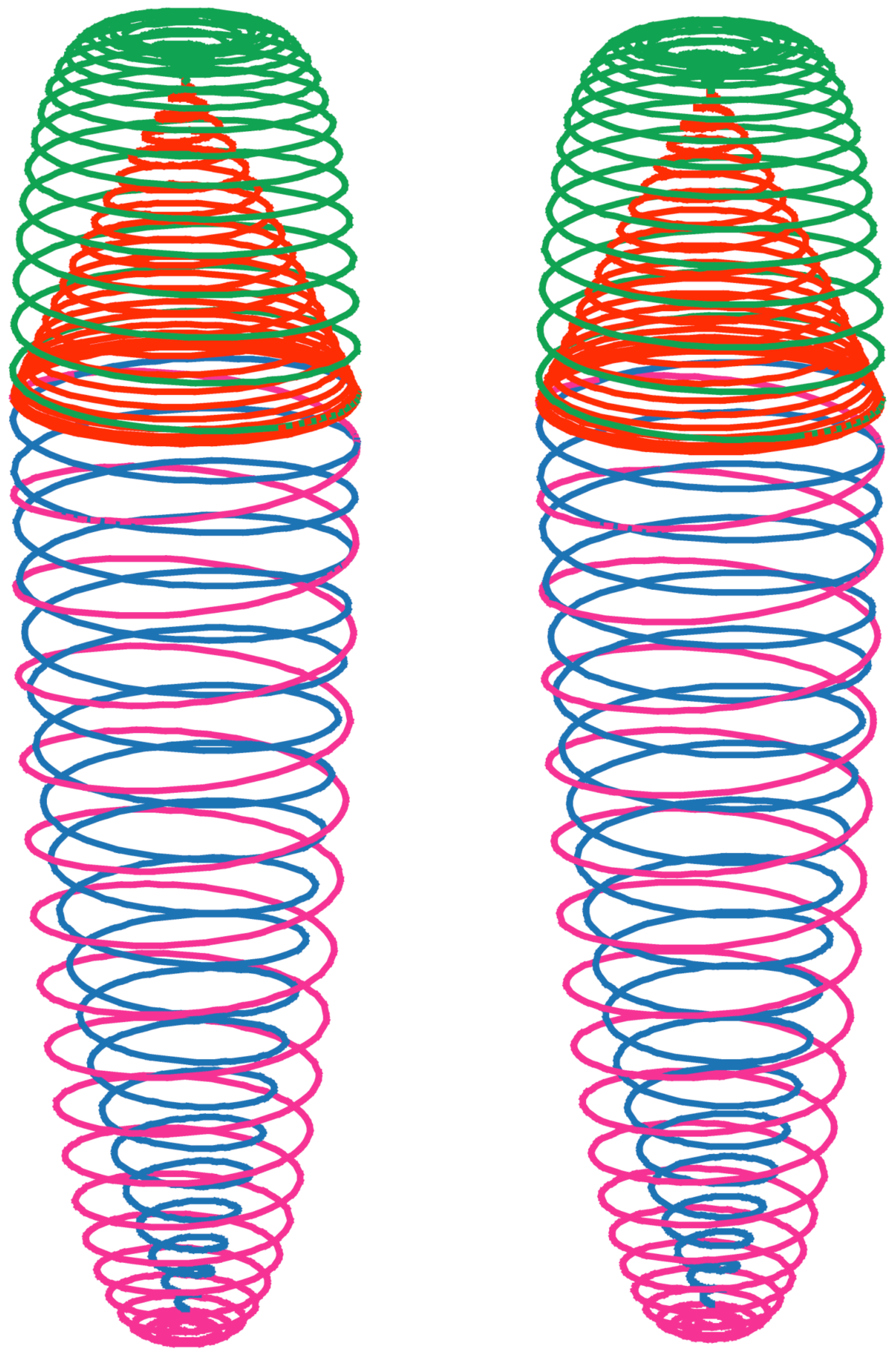

Figure 19. A stereo view of the possible agent helices. These helices classify the stabilities of a pair of agents as a function of action angle. The red fermion portion covers action angles θ *from 0 to* π, *where* θ = 0 *corresponds to the broad base of the red portion. The green boson portion covers action angles* θ *from* π *to* 2π. *The blue boson*

portion covers action angles θ *from* 2π *to* 3π. *The magenta fermion portion covers action angles* 3π *to* 4π. *The approach angle has one fixed value in this figure.*

We now study the agent dynamics as a Markov chain, and develop its transition probability. We consider first the dynamics of two agents, referring to Figure 20. We assume both agents have approach angle a and jump length l. Let $\boldsymbol{q}_\mathrm{i} \in R^3$, i = 1,2, be the current view of agent i, and write $\boldsymbol{q} = (\boldsymbol{q}_1, \boldsymbol{q}_2)$. Let $\boldsymbol{v}_\mathrm{i} \in R^3$ be the jump that took agent i to its current view, and write $\boldsymbol{v} = (\boldsymbol{v}_1, \boldsymbol{v}_2)$. Let $\boldsymbol{r}_\mathrm{i} \in R^3$, i = 1,2, be the next view of agent i, and write $\boldsymbol{r} = (\boldsymbol{r}_1, \boldsymbol{r}_2)$. Let $\boldsymbol{w}_\mathrm{i} \in R^3$ be the next jump of agent i, and write $\boldsymbol{w} = (\boldsymbol{w}_1, \boldsymbol{w}_2)$. Let $\boldsymbol{d}_\mathrm{i}$ denote the partner vector of agent i, e.g., $\boldsymbol{d}_1 = (\boldsymbol{q}_2 - \boldsymbol{q}_1) / |\boldsymbol{q}_2 - \boldsymbol{q}_1|$. Let $\boldsymbol{o}_\mathrm{i} \in R^3$, i = 1,2, be the orientation vector for the action plane of agent i, e.g., $\boldsymbol{o}_1 = (\boldsymbol{v}_1 \times \boldsymbol{d}_1) / |\boldsymbol{v} \times \boldsymbol{d}_1|$. Let $\boldsymbol{h}_\mathrm{i} = \boldsymbol{o}_\mathrm{i} \times \boldsymbol{d}_\mathrm{i}$, which we call the unit *heading vector.* Then the next jump of agent i will be the vector $l\ (\boldsymbol{h}_i\ cos(a) + \boldsymbol{d}_i\ sin(a))$. Let c_a denote $cos(a)$ and s_a denote $sin(a)$. Then the transition probability of the pair of agents is the Markovian kernel

$$
\begin{aligned}
P(\boldsymbol{q} \times \boldsymbol{v}, \boldsymbol{r} \times \boldsymbol{w}) \quad = \quad & (8\pi^3)^{-1} \det \Sigma^{-1} \\
& \exp[\, -(\boldsymbol{w}_1 - \mu_1)^T \Sigma^{-1} (\boldsymbol{w}_1 - \mu_1)\,] \\
& \exp[\, -(\boldsymbol{w}_2 - \mu_2)^T \Sigma^{-1} (\boldsymbol{w}_2 - \mu_2)\,] \\
& \varepsilon(\boldsymbol{q}_1 + \boldsymbol{w}_1, \boldsymbol{r}_1) \\
& \varepsilon(\boldsymbol{q}_2 + \boldsymbol{w}_2, \boldsymbol{r}_2),
\end{aligned}
$$

where Σ is a 3×3 covariance matrix, $\mu_\mathrm{i} = l\ (\boldsymbol{h}_\mathrm{i}\ c_a + \boldsymbol{d}_\mathrm{i}\ s_a)$, and $\varepsilon(\boldsymbol{x}_0, \boldsymbol{x})$ denotes Dirac measure at x_0.

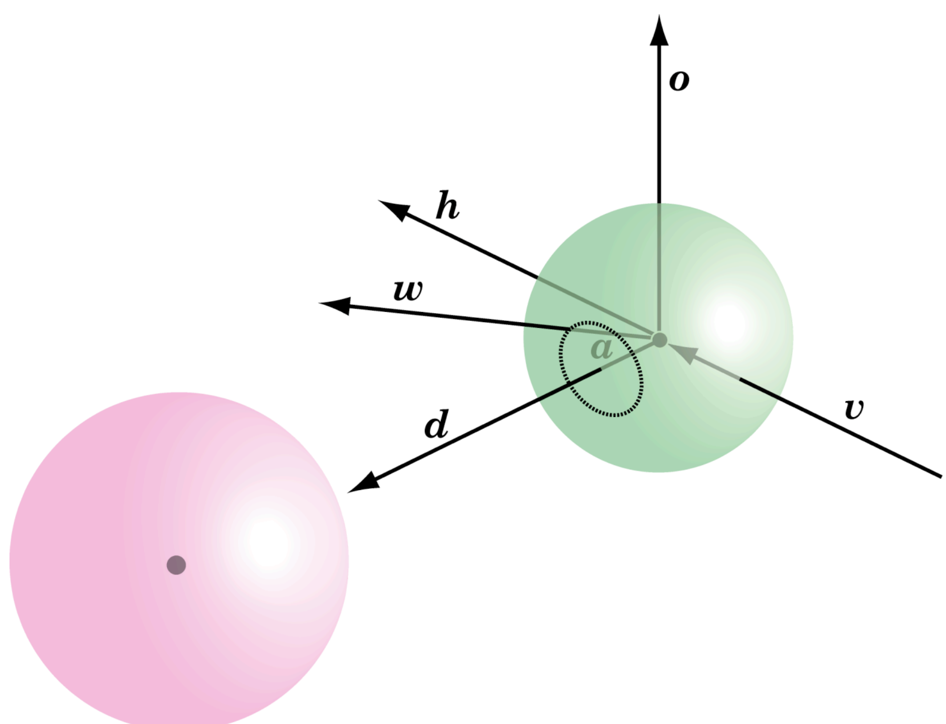

Figure 20. The geometry for the transition probability of a two-agent dynamics.

So far we have considered two agents in interaction. When we consider more agents, a new issue arises: Who looks, and at whom? For two agents we assumed that each agent looks at the other. For more agents, we assume the following:

1. Each agent observes one agent or no agent per look.
2. The probability that one agent looks at another falls monotonically with their distance in view-look space.
3. The looks of agents are chosen independently.

These assumptions govern the *look probability distribution*, or for brevity, the *look link.* In the simulations below, the look link is implemented as follows. At each step of the simulation, the distances between an agent and all other agents are computed. If there are *n* such distances, then *n* Gaussian-distributed random numbers are chosen. The random number whose magnitude most exceeds its corresponding agent distance determines which agent is observed. If no random number exceeds its corresponding agent distance, then no agent is observed. The mean of the Gaussian is zero and its variance is a free parameter to be adjusted based on simulation results.

Figure 21 shows the first 15 steps of a simulation with three agents, each having an approach angle of 10 degrees. Their initial positions and initial steps were chosen randomly from a uniform distribution. The three quickly form a stable boson unit, and head off together. Figure 22 shows a simulation with four agents, each having an approach angle of 5 degrees, forming a stable unit.

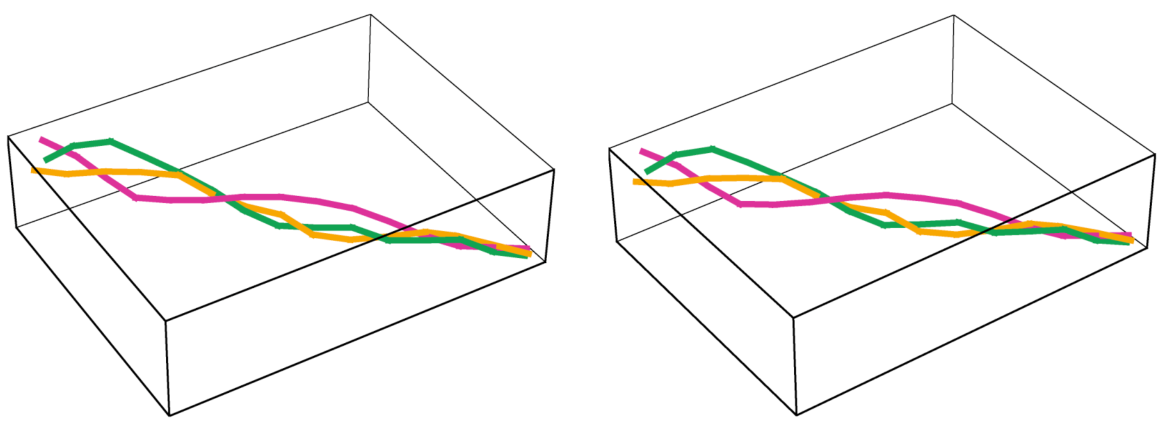

Figure 21. A stereo view of three agents forming a stable boson unit. Exponential look link.

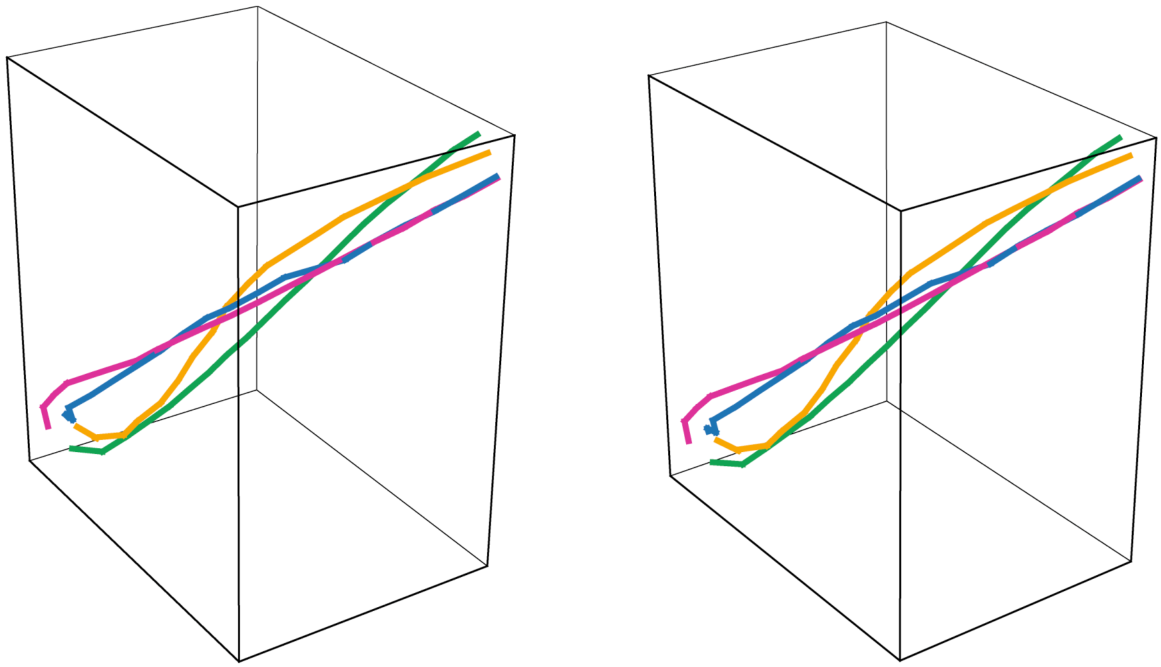

Figure 22. A stereo view of four agents forming a stable boson unit. Exponential look link.

The strength of the look link is critical to the stable units that are formed. The simulations of Figures 21 and 22 have look links governed by an exponential with a large mean. This is a *strong look link*. If we greatly reduce the mean of the exponential, i.e., if we use a weaker look link, we can get subgroups of agents to break off into separate units. This is illustrated in Figure 23, which shows a simulation having 8 agents, each having an approach angle of 10 degrees. Three pairs of agents break off and go their own ways. Two agents are left largely inert.

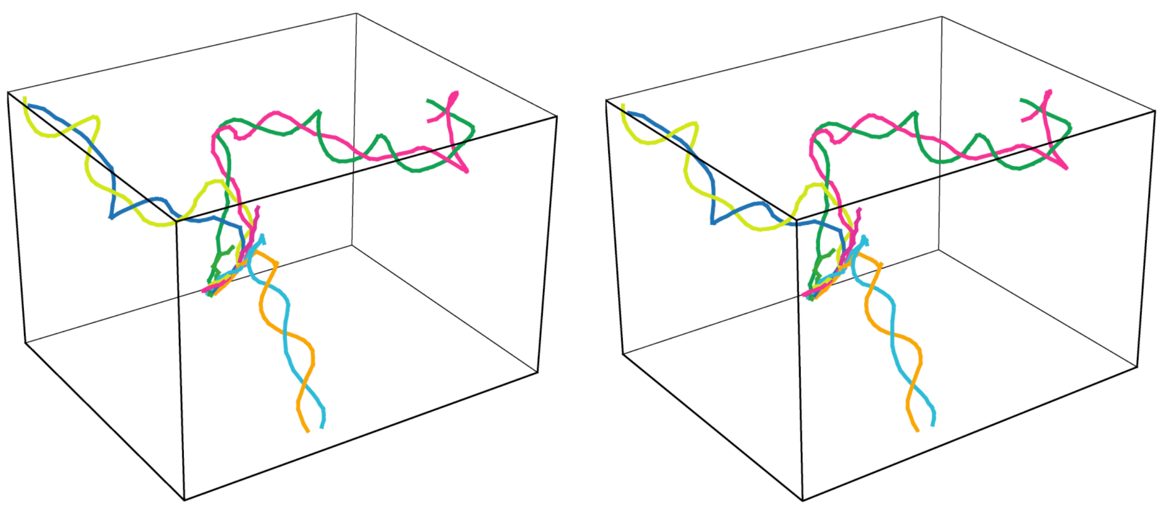

Figure 23. A stereo view of a simulation with eight agents, each having an approach angle of 10 degrees, and with a weak look link. Exponential look link.

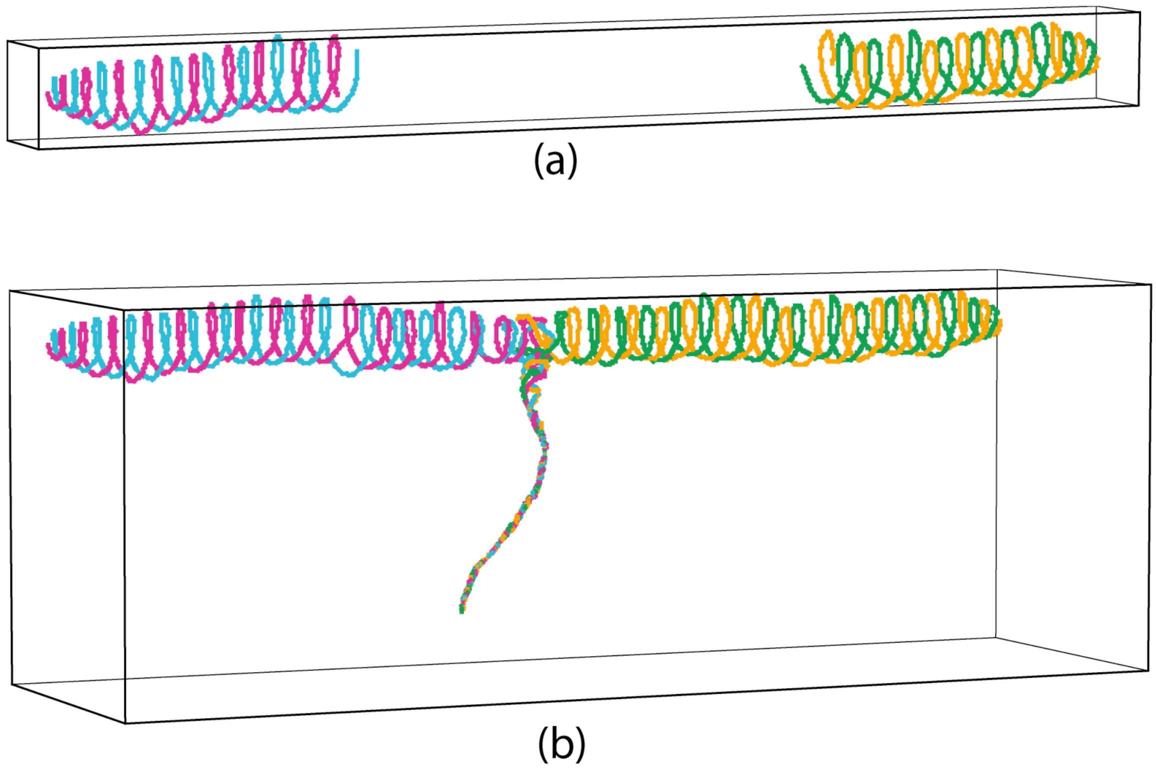

Figure 24. Particle accelerator experiment with two fermions. (a) *Two fermions with action angles of 10 degrees and approach angles of 10 degrees are launched at each other. 750 steps of the dynamics are shown.* (b) *This is the same dynamics as in (a), but now 1750 steps are shown. The fermions collide, and their four agents form a boson and exit.*

We now extend the agent dynamics to four or more view dimensions. This gives the agents more degrees of freedom in the views they can adopt. It also might help make contact with M theory in physics, where the membrane dynamics occurs in 10 spatial and 1 temporal dimension. All aspects of the agent dynamics in the higher-dimensional cases are as specified in the three-dimensional case. Specifically, each agent has a characteristic angle and length for its steps, and each agent steps in its action plane with a step whose dot product with its previous step is positive, as per the condition of consistent direction. The only difference is that we cannot use vector cross products to define the action planes and step vectors in higher dimensions. So we must use a more general formalism to specify the step vector. To derive this, we refer to Figure 25, which shows a top view of the action plane of an agent.

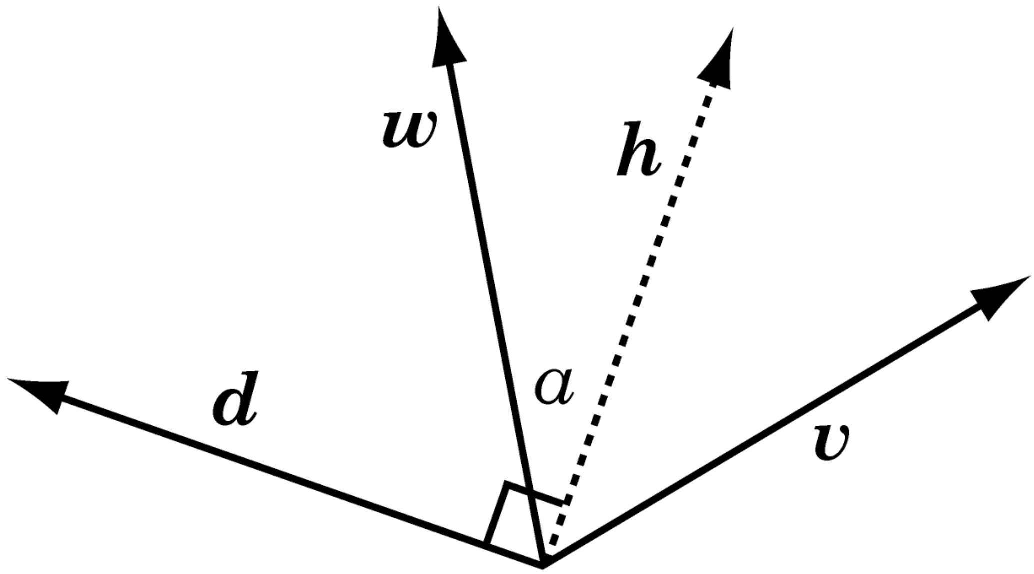

Figure 25. Top view of the action plane of an agent.. The previous jump is ***v.*** *The partner vector is* ***d****. The vector* ***h*** *is orthogonal to* ***d****. The next jump is* ***w****, with characteristic angle a.*

We are given $\boldsymbol{v}$ and unit vector $\boldsymbol{d}$, and wish to find the next jump $\boldsymbol{w}$. To this end, we first determine a unit vector $\boldsymbol{h}$ that is (1) orthogonal to $\boldsymbol{d}$, i.e., $\boldsymbol{h} \cdot \boldsymbol{d} = 0$, and (2) is in the same direction as $\boldsymbol{v}$, i.e., $\boldsymbol{v} \cdot \boldsymbol{h} \geq 0$. Jon Merzel found the solution $\boldsymbol{h} = [(\boldsymbol{d} \cdot \boldsymbol{d})\boldsymbol{v} - (\boldsymbol{v} \cdot \boldsymbol{d})\boldsymbol{d}]/|[(\boldsymbol{d} \cdot \boldsymbol{d})\boldsymbol{v} - (\boldsymbol{v} \cdot \boldsymbol{d})\boldsymbol{d}|$. The exception is the nongeneric case in which $\boldsymbol{v} \cdot \boldsymbol{d} = 0$, in which case we just set $\boldsymbol{h} = \boldsymbol{v}/|\boldsymbol{v}|$. Then the mean vector for the next jump is $\boldsymbol{w} = l\, cos(\mathrm{a})\, \boldsymbol{h} + l\, sin(\mathrm{a})\, \boldsymbol{d}$. Here, l is the characteristic length of the agent's jump.

Figure 26 shows four orthographic projections of the dynamics of four agents in a 4D view space. Each projection is obtained by deleting one of the four view dimensions, and displaying the projection of the dynamics into the other three dimensions.

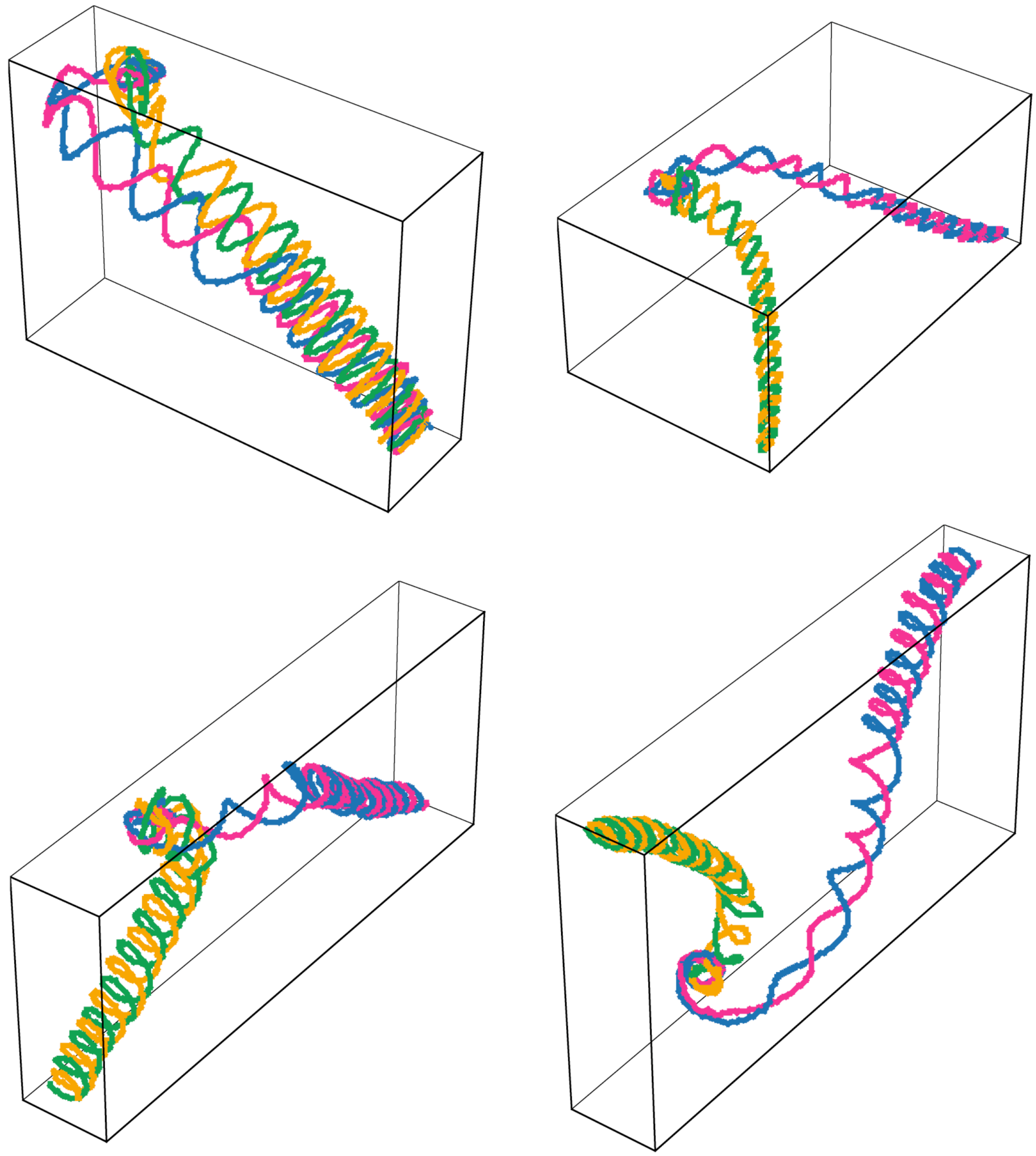

Figure 26. Four orthographic projections of the dynamics of four agents in a 4D view space.

Appendix.
Further issues to explore:

1. Agents interacting with different approach angles.
2. The transition probability for the 3D dynamics with other models of dispersion.
3. Anti-particles? Relativistic effect. Need to develop relativity aspects of theory.
4. Could two pairs of agents, each pair interacting like a boson, act together like a fermion? Could two bosons cycle around each other in a fermion dynamics?

5. Bosons can have mass. How does this arise in this theory?
6. Lorentz invariance
7. Relation between SU(2) double cover of SO(3) and the 4π action angle needed to classify two-agent dynamics. Relation to spin half. Is the entire 4π just for spin half fermions? If so, where do bosons come in? From pairing two fermions?
8. Look link and gravitation
9. Nonlocality, entanglement